

IMPACT PROTECTION SYSTEM WITH QUAZINULL RIGIDITY

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Impact action and intensive low-frequency oscillations arising during the mobile machineries' operation often have random character and are dangerous for the working personnel; also vibrations cause the breakage of mechanisms. As a rule, such impacts have unpredictable character and effective protection from such impacts is very urgent and still open problem.

The impact protection systems should reduce the level of impact pulse till safe margins guaranteeing proper industry objects' operation during dynamic impact and after it. Impact damping elements included in the composition of such protection means should provide smooth damping of impact energy during motion and also provide the system return into initial position after the impulse action completion. [1].

The experience shows that traditional vibroprotection means such as rubber and rubber-metal suspensions frequently do not provide the required parameters of vibration and impact decreasing. Rubber distinctly changes its elastic-damping properties with temperature variation and is wear susceptible under radiation influence and dissolves in chemically aggressive media. [2, 3].

So far there is no isolator to the same extent protecting from impacts and vibrations though this problem was set up long ago. [4] Besides, it's common that suspensions, designed for protection from vibrations don't provide the protection from long duration and high amplitude impacts, because considerable travel of protection system is required.

The possibility of quazinull rigidity (QR) elastic systems' application on the basis of Mesis frame ("systems with jump") for protection of dynamic objects was shown firstly by Alabuzhev P.M. in 1967 year. [5]

The relation of restoring force from displacement for Mesis frame looks like sine graph. The idea is in sorting out third vertical spring having deflection rate c , so that the inclination angle of $c \cdot x$ line matches with inclination angle of sine's linear part. Then the superposition of forces will give a region with QR. If we select corresponding mass, then it will be supported in stationary position by the QR spring.

QR systems find application in vibroprotection seats, train protection from vibration on railway and in some other technique fields [5, 6]. Although QR systems have obvious advantages, application of such systems still is not widespread.

One of the basic weaknesses of Alabuzhev's vibroprotection systems is small QR travel range (few cm), (picture 1).

Technology of engineering and production of principally new car suspensions of various types were invented by Ukraine engineering institute NANU and NKAU. The studies were based on the theoretical and experimental investigation of QR systems' operation range. The indicated suspensions have QR operation range of static characteristic (picture 1), the installation of hydraulic damper and satisfy the requirements of travel softness and firmness of high class automobiles.

Another disadvantage of this vibroprotection system is the lack of damping. The impact protection cannot be sufficiently effective.

Author proved the possibility of QR systems with operation range several times exceeding the existing QR systems' range. The suggested elastic systems' elastic elements are situated under definite angles and have corresponding lengths (picture 2).

Restoring force $F_{\Sigma}(x)$ of the system, composed of linear springs without account of friction is determined by the equation.

$$F_{\Sigma}(x) = F1(x) + F2(x), \quad (1)$$

where

$$F1(x) = 2 \cdot l_1 \cdot c_1 \cdot x / \sqrt{l_1^2 + x^2} - 2 \cdot l_1 \cdot x \cdot \cos \alpha_1 -$$

$$2 \cdot l_1^2 \cdot c_1 \cdot \cos \alpha_1 / \sqrt{l_1^2 + x^2} - 2 \cdot l_1 \cdot x \cdot \cos \alpha_1 + 2 \cdot c_1 \cdot l_1 \cdot \cos \alpha_1 - 2 \cdot c_1 \cdot x;$$

$$F2(x) = 2 \cdot l_2 \cdot c_2 \cdot x / \sqrt{l_2^2 + x^2} - 2 \cdot l_2 \cdot x \cdot \cos \alpha_2 -$$

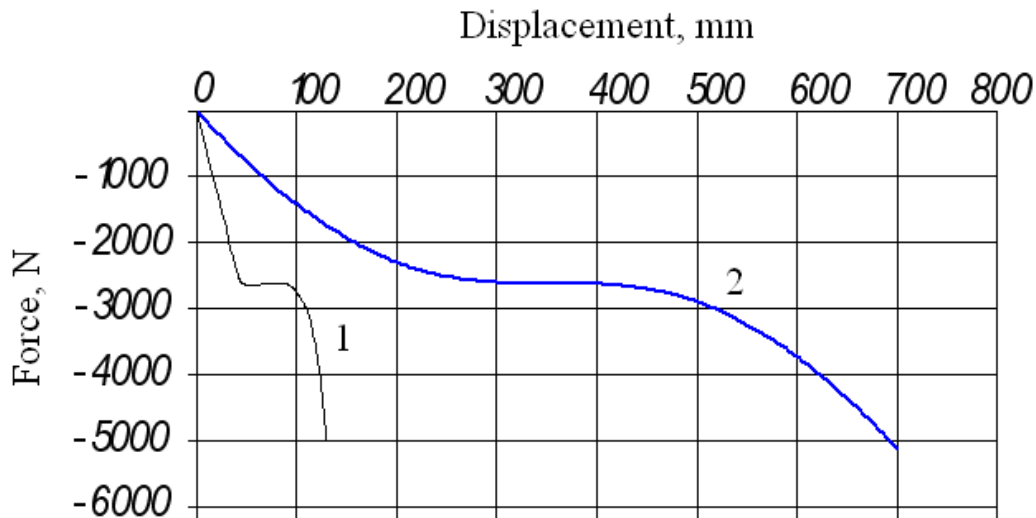
$$2 \cdot l_2^2 \cdot c_2 \cdot \cos \alpha_2 / \sqrt{l_2^2 + x^2} + 2 \cdot c_2 \cdot l_2 \cdot \cos \alpha_2 - 2 \cdot c_2 \cdot x;$$

$l_{1,2}$ – corresponding unstrained spring lengths (picture 2);

$c_{1,2}$ – corresponding spring deflection rates (picture 2);

$\alpha_{1,2}$ – corresponding unstrained spring inclination angles (picture 2);

x – the displacement of impact protected body from zero position.



Picture 1- The relation of restoring force from displacement for Alabuzhev's (curve 2) and NANU&NKAU (curve 1) systems

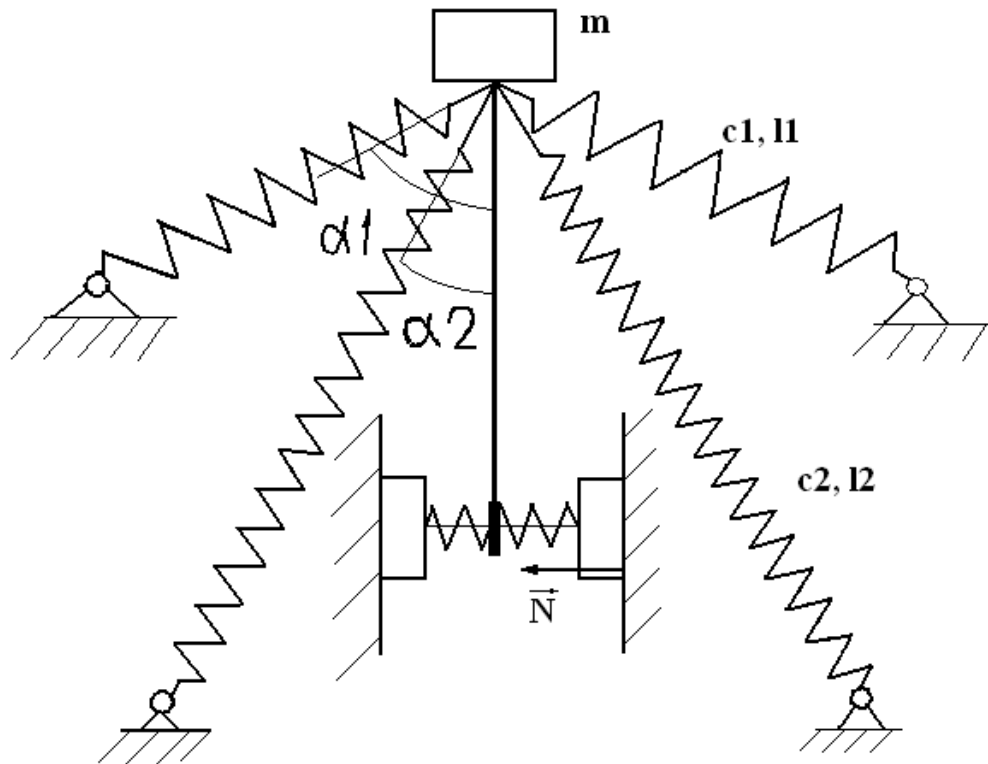
Generally, characteristics with QR range are possible for elastic systems that consist not only of linear springs but also from other elastic elements: pneumatic springs, rubber elements of circular cross-section and etc. [8].

An impact energy absorber on the principle of Coulomb's friction is illustrated on the scheme of vibroprotective system. A constant force ($b0/2$) is added to $F_{\Sigma}(x)$ at downward motion and the same force is subtracted in upward motion. This is shown on the picture 3. The constant $b0$ is determined by formula: $b0/2 = 2 \cdot f \cdot N$ (picture 2). The dependence of restoring force from displacement is shown on the picture 4. In this case, the impact protected body's mass is related with $b0$ by the following formula (picture 3):

$$m \cdot g = b0 / 2 \quad (2)$$

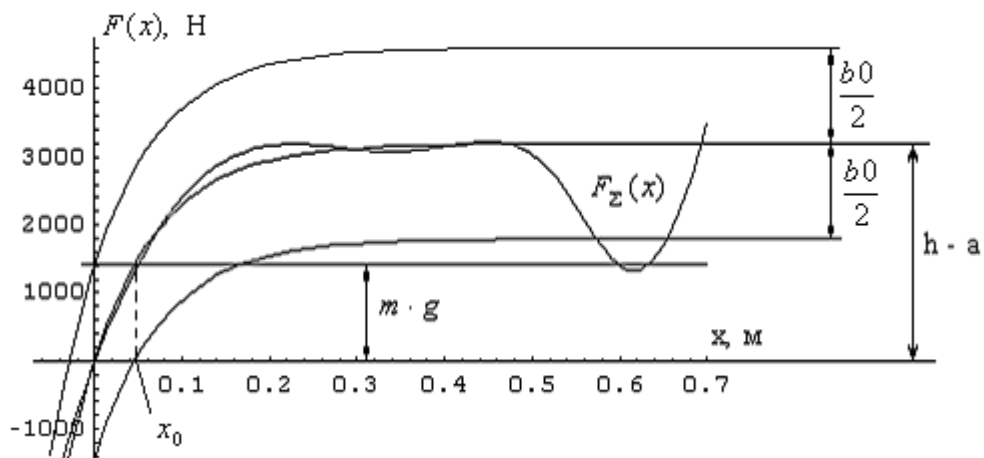
The dependence $F_{\Sigma}(x)$ is derived for the following system parameters:

$$l_1 = 0,60 \text{ m}; l_2 = 0,59 \text{ m}; c_1 = 57900 \text{ N/m}; c_2 = 6960 \text{ N/m}; \alpha_1 = 65^{\circ}; \alpha_2 = 12^{\circ}.$$



Picture 2 – Principal scheme of vibroprotection QR system at the presence of Coulomb's friction

The dependence $F(x) = F_z(x) + b/2 - m \cdot g = F_z(x)$ (picture 3) may be approximated by the function: $F_1(x) = h \cdot th(k \cdot x + d) - a$. The results of such an approximation are shown on the picture 4. Here: $h = 53500$; $k = 6,5$; $d = 1,74$; $a = 50302,12$. The values are given in SI system in this work.



Picture 3 – Relation of restoring force from displacement for a system, consisting of the linear springs at the presence of Coulomb's friction

Finally we have: the relation of restoring force from displacement is described by $F_1(x)$ function at downward, and by $F_*(x) = F_1(x) - b_0$ function at upward motion (picture 4).

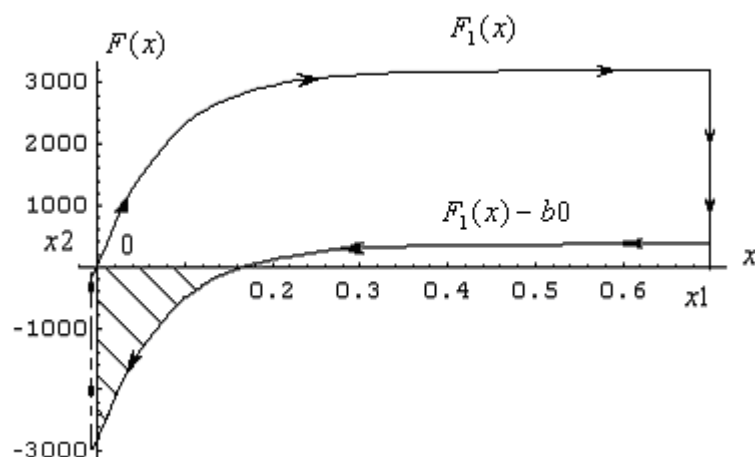
The impact protected mass m will gain V_0 velocity after the impact. When kinetic energy of the body $\frac{m \cdot V_0^2}{2}$ transfers into potential it will travel to the lower boundary position with x_1 coordinate (picture 4). Then the following condition should be satisfied.

$$\frac{m \cdot V_0^2}{2} = \int_0^{x_1} F(x) dx \quad (3)$$

After integration, taking into account that $F(x) = h \cdot th(k \cdot x + d) - a$, we get:

$$\frac{m \cdot V_0^2}{2} = \frac{h}{k} \cdot \ln\left(\frac{ch(k \cdot x_1 + d)}{ch(d)}\right) - a \cdot x_1 \quad (4)$$

Solving this equation numerically by dichotomy method x_1 is determined. For the data given above and $\frac{m \cdot V_0^2}{2} = 2000$ J: $x_1 = 0,703$ m.



Picture 4 – The dependence of restoring force from displacement in the form of two functions: $F(x) = h \cdot th(k \cdot x + d) - a$ (upper curve) and $F_*(x) = F(x) - b_0$ (lower curve)

Let's determine b_0 value (correspondingly mass m) from the condition of impact energy absorption in one oscillation. Evidently, $(h-a) > b_0/2$. For the case being observed in the work: $b_0 < 6395,76$. Besides, the conditions below should be hold (pictures 3, 4):

$$x_2 < 0; F_*(x_2) \approx F_1(x_1), \quad (4)$$

where x_2 - body coordinate at stop.

If x_2 coordinate will remain more than zero during upward motion (will get into shaded area – picture 4), then the impact protected body will not return into zero position.

The motion of the impact protected body under action of $F_1(x)$ and $F_*(x)$ forces (picture 4) is described by the next differential equations (DE).

$$m \cdot x_1'' = -F_1(x) \quad (5)$$

$$m \cdot x_2'' = -F_*(x) \quad (6)$$

Boundary conditions for the equations are the following:

- 1) $t = 0: x_1 = 0; x_1' = V_0;$
- 2) $t = \tau: x_2(\tau) = x_1(\tau); x_2'(\tau) = x_1'(\tau) = 0;$
- 3) $t = T: x_2'(T) = 0,$

where τ - the time period of motion until the first stop;

T – the time period of one oscillation.

For $x_2'(T) = 0$ hysteresis area corresponds to the energy, obtained after the impact:

$$(x_1 - x_2) \cdot b_0 = \frac{m \cdot V_0^2}{2} \quad (8)$$

This condition is equivalent to the impact energy absorbed during one oscillation. The solution of (5) and (6) DE was derived numerically by “Mathematics 5” computing package. The body mass m and correspondingly b_0 ($b_0 = 2 \cdot g \cdot m$) values were set. At the same time the condition (4) should be satisfied i.e. $x_2(T)$ coordinate should be less than zero (picture 4).

On the picture 5 the functional dependence body coordinate-time is depicted (numerical solution of formula (4)). The body will travel from neutral position into

lower boundary ($x_1=0,703$ m – the tangent point with line – picture 6). The body velocity this moment equals to zero.

The upward body motion is described by equation (6). The function of position and velocity in time are shown on pictures 7 and 8 taking into account the condition (7). As it is seen from this pictures $x_2 \equiv x_2(T)$ ($x_2'(T) = 0$) equals 0,05 m (pictures 4, 7).

Given b_0 value was derived from the condition that x_2 ($x_2 < 0$) should not be very small. It occurs that for $F_*(x_2) = F_1(x_1)$ coordinate $x_2=0,003$ m.

For random variation of impact energy the body may not return into neutral position (will fall into the shaded area – picture 4). b_0 value was decreased in order to get certainly $x_2=-0,05$ m. At that $F_*(x_2) = 4300$ N (picture 9), then

$$\left| \frac{F_*(x_2)}{h-a} \right| = \frac{4300}{3197,88} = 1,34. \text{ i.e. in maximum upward deviation from neutral position}$$

the body experience 1,34 times more absolute value force, than in maximum deviation downward. b_0 value is equal to 2805,57 N. Correspondingly: $m = b_0/(2 \cdot g) = 142,85$ kg.

If the impact energy is not changing through a certain technological process (here: $\frac{m \cdot V_0^2}{2} = 2000$ J), then the obtained result seems very fair. Impact energy will be absorbed in one oscillation. Modern suspensions have 50-60 % of dissipation work from overall absorbed energy in stroke.

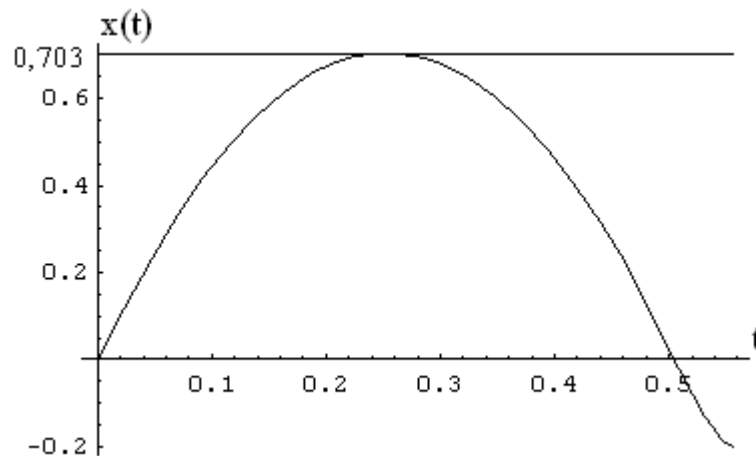
It's a rare situation in technique, when impact energy is not changing through a certain technological process. That's why restoring force's characteristic, shown on picture 4, is not optimal. More perspective is the type of characteristic plot on picture 10. Generally, there is no zone here, when in downward travel the restoring force is also directed downward (it will not return into zero position at body's halt in that area). This is achieved by variable friction. It is equal to zero in zero position. Then, it is increased in downward motion and is const beginning from AB section. The reactions appearing during elastic elements' motion along inclined planes are not accounted in this case. It is assumed that there can be accomplished their smallness in comparison with friction force (by increasing friction coefficient) and restoring force.

Let the friction force be calculated by the following formula:

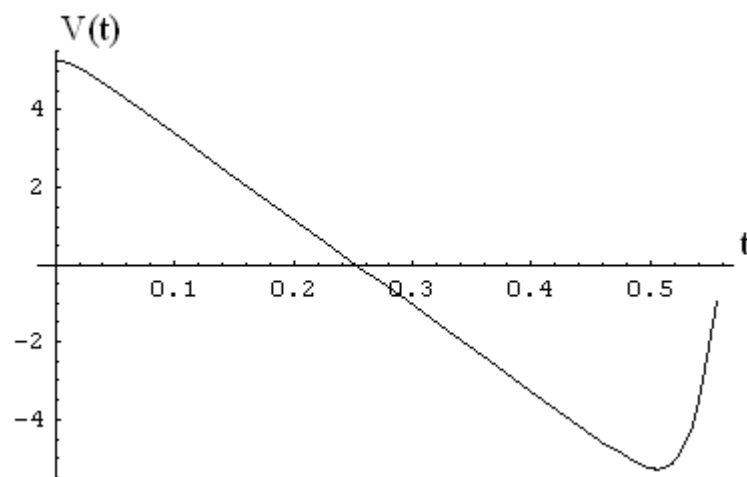
$$b(x) / 2 = b_0 \cdot x / (2 \cdot x_*), \quad (9)$$

where b_0 - the value, derived for the case with const friction;

x_* - AB section coordinate (picture 10). Let's take $x_* = 0,35\text{m}$.



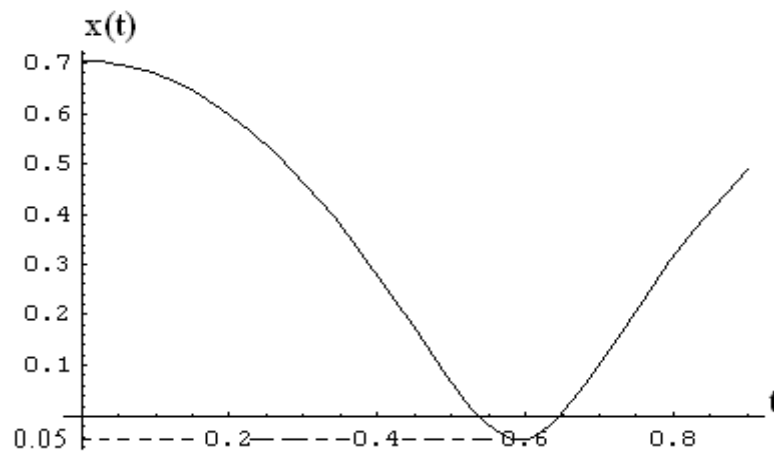
Picture 5. Relation of body coordinate (that is calculated by formula (5): $m \cdot x_1'' = -F_1(x)$ with account of condition (7) depending on the time.



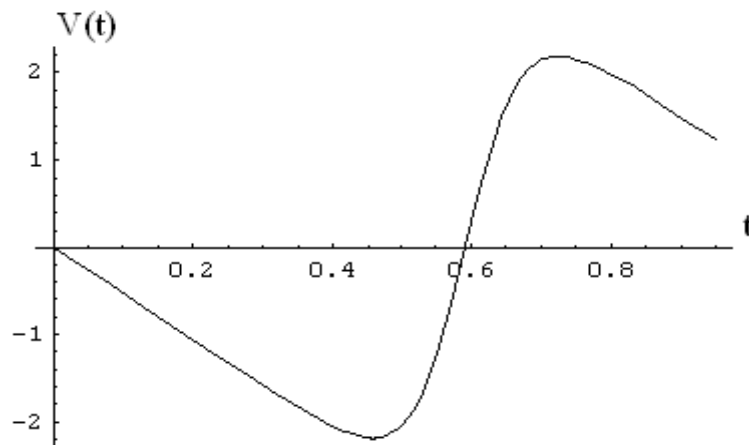
Picture 6 – Relation of velocity (calculated by formula (5): $m \cdot x_1'' = -F_1(x)$ with account of condition (7) depending on the time

It is technically not difficult to provide a varying force of friction, bounded by the formula (9). On the picture 11 one of the device's scheme variants that maintains prescribed law of friction variation. Spring 1 is unstrained and friction force is equal to

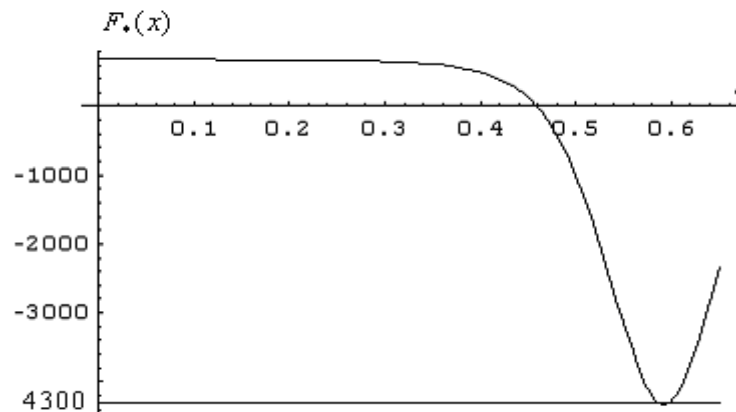
zero in section 0-0. At downward travel spring will shrink by the law, determined by inclination guide 2. Springs press the friction disks 3 to the guide 4. Friction disks are rigidly jointed with impact protected body 5. At downward body motion and reaching x_* coordinate, friction force will be constant and equal to $b_0/2$ (9).



Picture 7. Relation of body coordinates (calculated by formula (6): $m \cdot x_2'' = -F_*(x)$ with account of condition (7) from time



Picture 8. Relation of body velocity (calculated by formula (6): $m \cdot x_2'' = -F_*(x)$ with account of condition (7) from time



Picture 9. Relation of restoring force $F_*(x)$ from time

Relation of restoring force from displacement, shown by dotted line on the picture 10 is described by following formula:

$$F(x) = h \cdot th(k \cdot (x + x_0) + 1.74) - a - m \cdot g, \quad (10)$$

where $x_0 = 0,045$ m (picture 3) – coordinate, that should be inserted to have $F(0) = 0$.

Downward travel until AB section (picture 10) will be described by the following DE:

$$m \cdot x_1'' = -h \cdot th(k \cdot (x + x_0) + d) + a - m \cdot g + b_0 \cdot x / (2 \cdot x_*), \quad (11)$$

where $m = 142,85$; $h = 53500$; $k = 6,5$; $d = 1,74$; $a = 50302,12$; $b_0 = 3366,69$; $x_* = 0,35$; $g = 9,82$.

Here b_0 is found from the condition that c_2 value (restoring force at upward motion after AB section - picture 10) should have small absolute value ($c_2 = h - a - m \cdot g - b/2 = 111,744$ N). Solving this equation numerically we get that for $x_1 = x_* = 0,35$ m, body velocity will be: $V_1^{AB} = 4,12$ m/s.

After reaching AB section the motion is described by other DE:

$$m \cdot x_1'' = -h \cdot th(k \cdot (x + x_0) + d) + a - m \cdot g + c_1, \quad (12)$$

where c_1 - restoring force value for $x > x_*$ (picture 10),

$$c_1 = h - a - m \cdot g + b/2 = 3478,43 \text{ N.}$$

The initial conditions for this DE will be the data obtained from the previous DE solution: $x_1 = 0,35$ m, $V_1^{AB} = 4,12$ m/s.

Body coordinate at halt will be: $x_1 = 0,7$ m. These data will be initial conditions for the next DE which describes the travel upward after halt until the section AB.

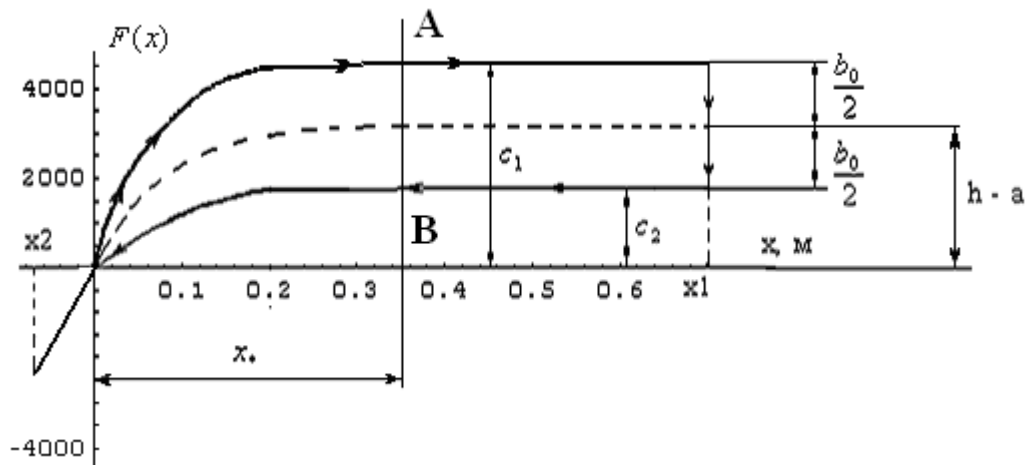
$$m \cdot x_2'' = -h \cdot th(k \cdot (x + x_0) + d) + a - m \cdot g + c_2 \quad (13)$$

When AB section is reached ($x_2 = 0,35$ m) the body velocity $V_{AB} = -0,74$ m/s. These data will be initial conditions for the next DE, which describes the travel from AB section until zero position.

$$m \cdot x_2'' = -h \cdot th(k \cdot (x + x_0) + d) + a - m \cdot g - b_0 \cdot x / (2 \cdot x_*) \quad (14)$$

The velocity equals to $V_{AB} = -1,82$ m/s when body reaches zero position. These data will be initial conditions for the next DE, which describes the motion from origin till halt (where friction force is absent).

$$m \cdot x_2'' = -h \cdot th(k \cdot (x + x_0) + d) + a - m \cdot g \quad (15)$$



Picture 10 – Relation of restoring force from the displacement in the form of two functions: $F(x) = h \cdot th(k \cdot (x + x_0) + d) - a + \frac{b}{2}$ (upper curve) and $F_*(x) = F(x) - b$, where b – alternating variable (restoring force without friction is shown by dotted line)

When the body stops ($x_2' = 0$) the body coordinate: $x_2 = -0,11$ m. Restoring force for this coordinate is equal: $F(-0,11) = -h \cdot th(k \cdot x + d) + a - m \cdot g = -5450$ N.

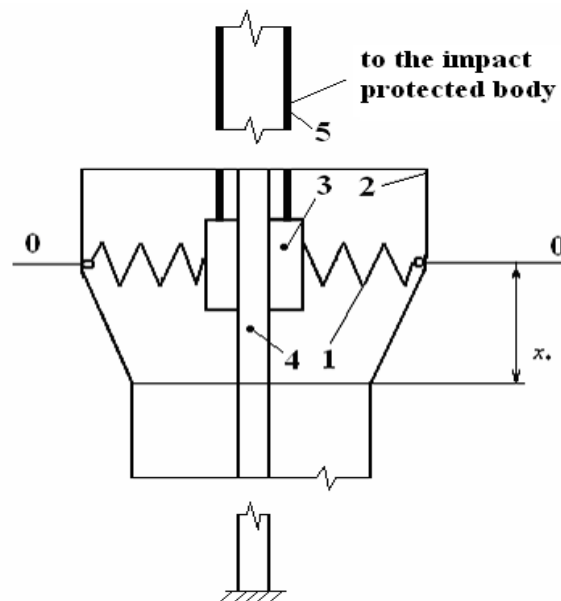
If by analogy with the previous case we calculate the relation of that value to the maximum restoring force for downward travel, then we get 1,56, i.e. for maximum upward deviation from zero position the impact protected body experience 1,56 times more force by absolute value, than for maximum downward deviation.

Let's compute the percentage of energy absorption during one period of oscillation for the given case (coefficients of energy absorption K_n):

$$K_n = \frac{\frac{m \cdot V^2}{2} - S}{\frac{m \cdot V^2}{2}}, \text{ where } \frac{m \cdot V^2}{2} = 2000 \text{ J; } S - \text{ the hysteresis loop's area (picture 10).}$$

For this case the hysteresis area (numerically calculated) is equal to: $S=234,7 \text{ J}$. Hence, $K_n = 0,88$, that is very good result for passive impact protection. The advantage of the last case (picture 10) as compared with the previous (picture 4) is the returning of body into zero position whenever impact energy changes.

When the kinetic energy of impact protected body is increased 2 times ($\frac{m \cdot V^2}{2} = 4000 \text{ Дж}$) and all other parameters are left the same; the repetition of all calculation chain – the solution of DE (11-15) – we get: $x_2 = -0,117 \text{ m}$; $F(-0,117) = -6000 \text{ N}$.

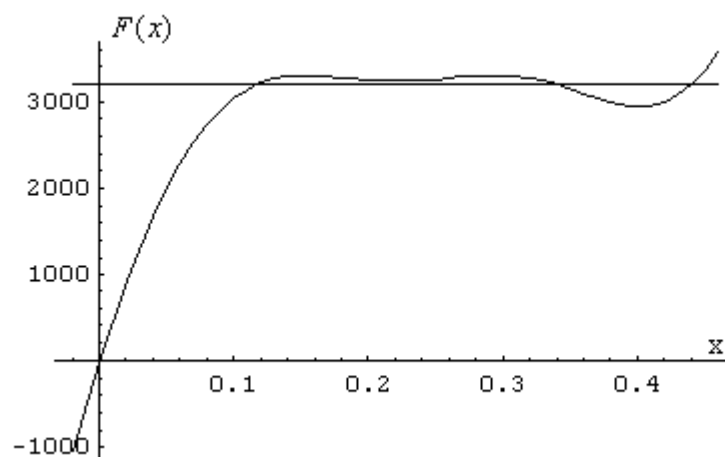


Picture 11 – Principal scheme of the device for production of varying Coulomb friction obtaining

When the kinetic energy of impact protected body is increased 2 times ($\frac{m \cdot V^2}{2} = 4000 \text{ Дж}$) and all other parameters are left the same; the repetition of all calculation chain – the solution of DE (11-15) – we get: $x_2 = -0,117 \text{ m}$; $F(-0,117) = -6000 \text{ N}$.

When the kinetic energy of the impact protected body is decreased two times ($\frac{m \cdot V^2}{2} = 1000 \text{ J}$) the solution of DE (11-15) we get: $x_2 = -0,105 \text{ m}$; $F(-0,105) = -4900 \text{ N}$. It's evident from these examples that kinetic energy has small changes, correspondingly, force $F(x_2)$ changes slightly also. This is evident advantage of the second type of impact protection.

As it is seen from picture 3 real relation $F_{\Sigma}(x)$ has a depression. When the left depression boundary is reached there is going to be a “jump” to the right depression boundary. This is undesirable. As a result of this, were found other system parameters shown on the picture 2 (l1; l2; c1; c2; a1; a2), that allow to get characteristics with QR zone. Such a characteristic is demonstrated on the picture 12. Although, the QR zone is less, the depression is considerably less.



Picture 12 – Relation of the restoring force from displacement without steeply-falling zone, which was shown on picture 3.

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