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ANALYSIS OF PIPELINE STRESS-STRAIN STATE IN SEABED LAYING

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Abstract. *Analysis of a pipeline bending in laying on a rigid seabed is carried out. The pipeline is modeled by a semi-infinite hinged elastic beam. Length of its sagging part is not known and is defined in the result of calculations. A formula for hydrostatic loading of a rod is derived in the paper; it is shown that in some cases simplified accounting of the loading by reduction of the rod's weight for the weight of the liquid superseded leads to significant errors. The seabed reaction is found for two rod models: the classical and Timoshenko ones. Analytical expressions are received and internal forces, moments, and stresses arising in the pipeline are shown depending on the distance from the seabed and liquid pressure. The pipeline's preliminary tension is considered.*

Keywords: *pipeline laying, elastic rod, hydrostatic loading, seabed reaction, asymptotic solution*

Introduction

Construction of sea gas pipelines is presently carried out with high intensity. Safety of people, environment, and equipment is one of the primary goals of the pipeline construction and maintenance. Damage to the integrity of a pipeline can lead to gas leakage and its further ignition. High safety standards pose high engineering requirements to the design, which stipulates the relevance of researches on the stress-strain analysis of a sea pipeline.

Damage of a pipeline can occur already in the process of its laying down on the sea bottom from a vessel. The pipeline is bent by its weight, and stresses in it can exceed allowable values [3]. A similar danger emerges in the course of raising of the pipeline from the bottom, in case if a discontinuity of laying has occurred, e.g., during a storm.

As the length of a pipeline sufficiently exceeds the size of its cross-section, the theory of rods can be put in the basis of calculations. Unfortunately, a simplified accounting of the hydrostatic loading on the rod predominates in the literature [4, 6, 7]; in some cases, this approach leads to significant errors. The correct formula remains known quite limitedly [5, 8].

In this paper, an alternative derivation of the correct formula based on Archimedes's principle is presented and comparison with the commonly known approach is carried out. The seabed is assumed rigid. Its reaction is found for two rod models – the classical and Timoshenko ones.

1. Classical Bernoulli-Euler beam

The problem of pipeline bending is considered in linear setting. The pipeline is modeled by a semi-infinite hinged elastic beam. The distance h between the hinge and the bottom is known (Fig. 1). The weight-loaded beam is also loaded by the pressure of the liquid. The length of its sagging part l is unknown and must be found. For a while, impact of the liquid is taken into account in the traditional way, i.e., through reduction of the beam's weight for the weight of the liquid superseded. The straight unstressed status of a beam is assumed reference configuration.

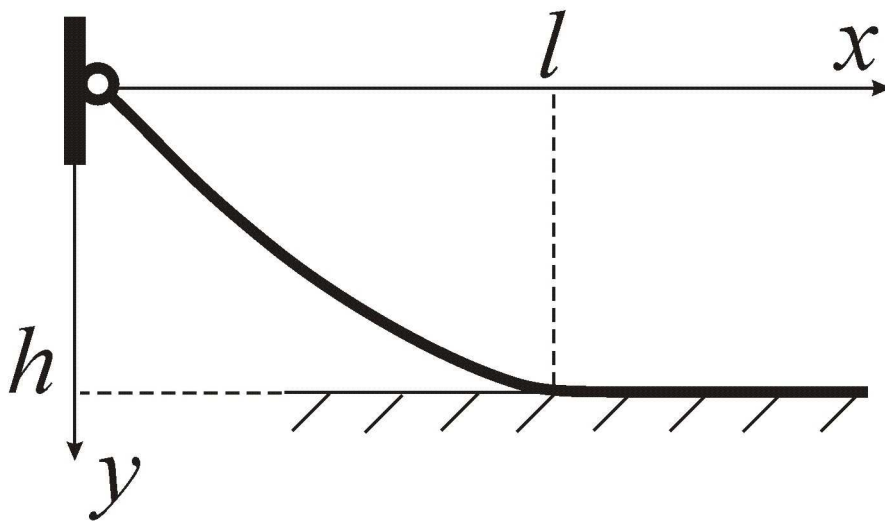


Fig. 1. A beam on the rigid seabed

In the classical model, rotation of beam's cross-sections $\theta(x)$ is connected with the deflection $u(x)$; balance equations and elastic stress-strain relations lead to the equation of the fourth order:

$$Q' + q = 0, \quad M' + Q = 0, \quad M = a\theta', \quad \theta = u' \Rightarrow a\partial^4 u = q, \quad (1)$$

where a is the bending stiffness of the beam; $\partial \equiv (\dots)'$ is differentiation with respect to x ; Q and M are the shear force and bending moment correspondingly; q is the external force per unit of the beam's length.

At $x < l$, the weight of the beam in a liquid per unit of length is $q = w = const$; with integrating relationships (1) and boundary conditions $u(0) = 0$, $M(0) = 0$, we shall receive

$$\begin{aligned} Q &= -wx + Q_0, \quad M = \frac{w}{2}x^2 - Q_0x, \quad \theta = a^{-1}\left(\frac{w}{6}x^3 - \frac{Q_0}{2}x^2\right) + \theta_0, \\ u &= a^{-1}\left(\frac{w}{24}x^4 - \frac{Q_0}{6}x^3\right) + \theta_0x. \end{aligned} \quad (2)$$

At the section of the fit $x > l$, the displacement is set: $u \equiv h \Rightarrow q = 0$. Hence, there is a concentrated force at the starting point:

$$q(x) = -R\delta(x-l) \quad \Rightarrow \quad Q|_{l-0}^{l+0} = R = wl - Q_0. \quad (3)$$

At this point, functions u , θ , and M are continuous:

$$\frac{w}{2}l^2 - Q_0l = 0, \quad a^{-1}\left(\frac{w}{6}l^3 - \frac{Q_0}{2}l^2\right) + \theta_0 = 0, \quad a^{-1}\left(\frac{w}{24}l^4 - \frac{Q_0}{6}l^3\right) + \theta_0l = h.$$

This is a system for determination of the force Q_0 and rotation θ_0 at the hinge, the length l and force R :

$$Q_0 = R = \frac{wl}{2}, \quad \theta_0 = \frac{wl^3}{12a}, \quad h = \frac{wl^4}{24a}. \quad (4)$$

At the section of the fit, the external distributed load is the sum of the beam's weight and contact pressure; therefore, the last one can be found by the formula

$$P_{cont} = w + R\delta(x-l). \quad (5)$$

2. Timoshenko beam

Let us consider a more elaborate model of the beam in which the deflection and rotation of cross-sections are independent. In system (1) the first three equations remain the same, while the fourth is replaced with the following [1]:

$$Q = b(u' - \theta), \quad (6)$$

where b is the shift stiffness. In (2), the first three relationships remain, and an additive is added in the expression for u :

$$u = a^{-1} \left(\frac{w}{24} x^4 - \frac{Q_0}{6} x^3 \right) + \theta_0 x + b^{-1} \left(-\frac{w}{2} x^2 + Q_0 x \right). \quad (7)$$

To find the contact pressure at the section of the fit, let us transform the equations:

$$b(u'' - \theta') + q = 0, \quad a\theta'' + b(u' - \theta) = 0, \quad \Leftrightarrow \begin{pmatrix} b\partial^2 & -b\partial \\ b\partial & a\partial^2 - b \end{pmatrix} \begin{pmatrix} u \\ \theta \end{pmatrix} = \begin{pmatrix} -q \\ 0 \end{pmatrix}. \quad (8)$$

Formally, this is a linear algebraic system; it can be solved through determinants:

$$Du = D_u = \begin{vmatrix} -q & -b\partial \\ 0 & a\partial^2 - b \end{vmatrix} \Rightarrow a\partial^4 u = q - \Lambda^{-2} \partial^2 q; \quad \Lambda^2 \equiv b/a. \quad (9)$$

Here from, considering the identity $u(x) \equiv h$, we can receive a distribution function of the total external loading

$$x > l: \quad q = q_0 e^{-\Lambda \xi} \quad (\xi \equiv x - l) \quad (10)$$

instead of the concentrated force (3).

Let us integrate the system of the four equations of such a loading:

$$Q = Q_1 + \frac{q_0}{\Lambda} e^{-\Lambda \xi}, \quad M = M_1 - Q_1 \xi + \frac{q_0}{\Lambda^2} e^{-\Lambda \xi}, \quad \theta = \theta_1 + a^{-1} \left(M_1 \xi - \frac{Q_1}{2} \xi^2 - \frac{q_0}{\Lambda^3} e^{-\Lambda \xi} \right), \quad (11)$$

$$u = u_1 + \theta_1 \xi + a^{-1} \left(\frac{M_1}{2} \xi^2 - \frac{Q_1}{6} \xi^3 + \frac{q_0}{\Lambda^4} e^{-\Lambda \xi} \right) + b^{-1} \left(Q_1 \xi - \frac{q_0}{\Lambda^2} e^{-\Lambda \xi} \right).$$

The last one is identically equal to h ; therefore,

$$Q_1 = 0, \quad M_1 = 0, \quad \theta_1 = -b^{-1} Q_1 = 0, \quad u_1 = h. \quad (12)$$

It is necessary to consider the continuity of Q , M , θ , and u at $x = l$ ($\xi = 0$):

$$-wl + Q_0 = \frac{q_0}{\Lambda}, \quad \frac{w}{2} l^2 - Q_0 l = \frac{q_0}{\Lambda^2}, \quad \frac{w}{6} l^3 - \frac{Q_0}{2} l^2 + a\theta_0 = -\frac{q_0}{\Lambda^3}, \quad (13)$$

$$a^{-1} \left(\frac{w}{24} l^4 - \frac{Q_0}{6} l^3 \right) + \theta_0 l + b^{-1} \left(-\frac{w}{2} l^2 + Q_0 l \right) = h.$$

This is the system for determination of Q_0 , θ_0 , q_0 , and l :

$$Q_0 = \frac{wl}{2} \frac{2+\beta}{1+\beta}, \quad \theta_0 = \frac{wl^3}{12a} \frac{2+(2+\beta)^2}{\beta(1+\beta)}, \quad q_0 = -\frac{w}{2} \frac{\beta^2}{1+\beta},$$

$$h = \frac{wl^4}{24a} \frac{\beta^3 + 5\beta^2 + 12(1 + \beta)}{\beta^2(1 + \beta)}, \quad \beta \equiv \Lambda l. \quad (14)$$

In case of a big value of the shift stiffness ($\beta \rightarrow \infty$), we come to formulas (4).

Distribution of the contact pressure at the section of the fit for the Timoshenko beam

$$P_{cont} = w - q_0 e^{-\Lambda \xi} \quad (15)$$

is more close to the real one as compared to the distribution for a classical beam (5).

Graphs of the contact pressure for both models are presented on Fig. 2.

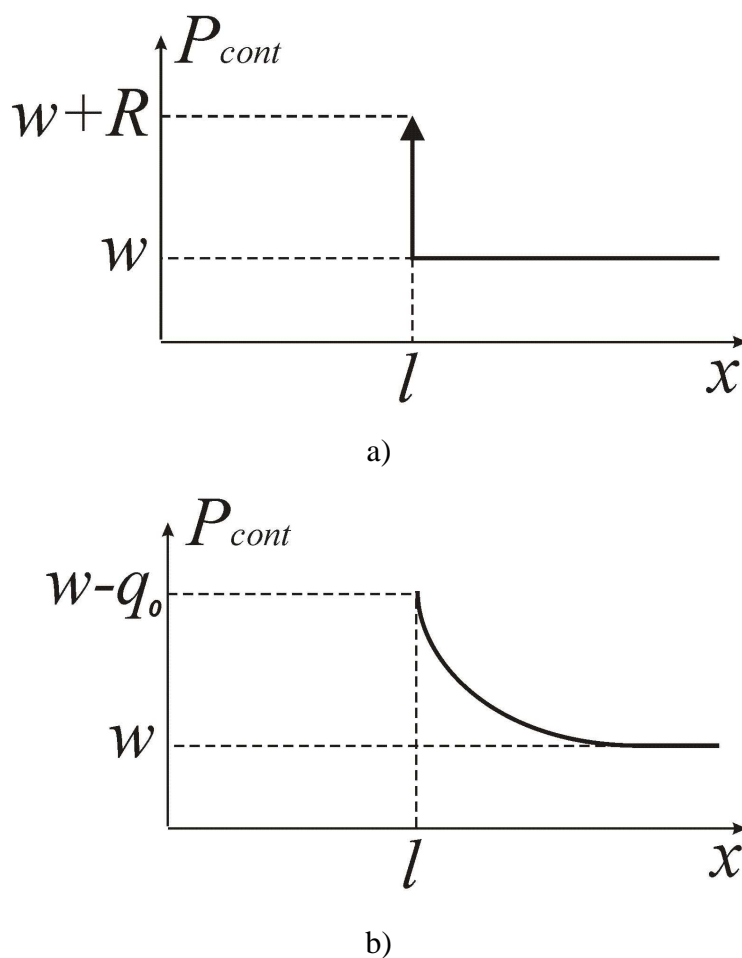


Fig. 2. The distribution of contact pressure:
(a) classical model; (b) Timoshenko model

3. Hydrostatic loading of a rod

There is no doubt that the resultant force impacting a completely submerged in a liquid rod equals to the weight of the liquid superseded by it. However, it would not be correct to determine the hydrostatic loading per unit of the rod length by simple division of the Archimedes's force by the length of the rod.

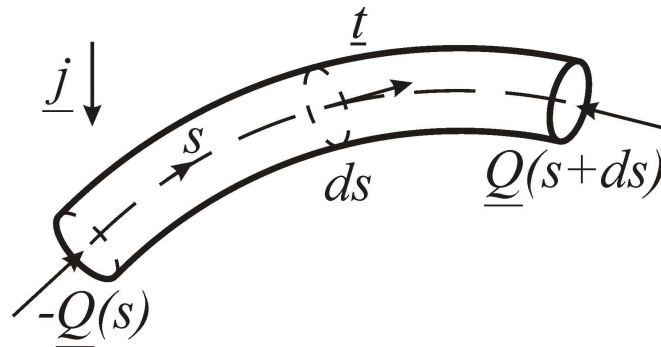


Fig. 3. An element of the rod

Let us bring into consideration the arc coordinate along the deformed axis of the rod s . Let us write down an equation of the balance of forces for a cut out piece of the rod of a ds length (Fig. 3); the distributed loading together with the forces at the end faces equals to the Archimedes's force:

$$\underline{q}ds - \underline{Q}(s) + \underline{Q}(s + ds) = -\gamma F ds \underline{j}, \quad (16)$$

where F is the cross-section area; γ is the weight of the liquid per unit of the volume.

Forces of the liquid at the end faces we will find by integration of the pressure on the cross-sectional area of the rod:

$$\begin{aligned} -\underline{Q}(s) &= -\underline{n}(s) \int_F p(y) dF = Fp[y(s)]\underline{t}(s), \quad \underline{Q}(s + ds) = -Fp[y(s + ds)]\underline{t}(s + ds), \Rightarrow \\ &\Rightarrow -\underline{Q}(s) + \underline{Q}(s + ds) = -(\underline{pt})' F ds, \end{aligned} \quad (17)$$

where p is the liquid pressure, \underline{n} is the unit normal vector to cross-section, \underline{t} is the unit tangent vector to the rod axis, and $(...)'$ means differentiation with respect to s .

From (16) and (17), we receive an expression for the hydrostatic force impacting a unit of the rod length:

$$\underline{q} = F \left[-\gamma \underline{j} + (\underline{pt})' \right]. \quad (18)$$

Let us remark here that in the commonly known formula (used in sec. 1, 2) the second summand taking into consideration the pressure difference in the liquid and the curvature of the deformed rod axis is missing.

4. A classical beam in a liquid

Let us consider the problem of bending of a classical beam in a liquid with consideration of the hydrostatic loading by formula (18).

As earlier, we will consider x as the arc coordinate in the reference straight configuration, and s in the deformed state. Let us write down expression for the radius vector of a rod particle:

$$\begin{aligned} \underline{r} &= x\underline{i} + u(x)\underline{j}; & \dot{\underline{r}}(s) &= \underline{t} = \dot{x}(\underline{i} + u'\underline{j}), \\ |\underline{t}| &= 1 = \dot{x}\sqrt{1+u'^2}, & \Rightarrow \dot{x} &= 1 - \frac{1}{2}u'^2 + \dots, \end{aligned}$$

where $(\dots)'$ and $(\dots)\dot{}$ denote differentiation with respect to x and s correspondingly;

\underline{i} and \underline{j} are the unit vectors of axes x and y .

Let us further on assume that the pressure difference in the liquid is caused by its weight only:

$$p = p_0 + \gamma u,$$

p_0 is the liquid pressure at the $y = 0$ hinge level.

Then we can find the hydrostatic force per unit of the beam length in the deformed state by formula (18):

$$q_y = F \left[\gamma(-1 + u'^2 \dot{x}^2) + p(u'' \dot{x}^2 + u' \ddot{x}) \right] = F \left[-\gamma + p_0 u'' + \gamma(uu')' \right] + \dots; \quad (19)$$

here the terms of the highest order are not written out as the beam deflections are small. However, the retention of the last summand in (19) is obligatory for taking into account the dependence of pressure in the liquid upon the value of the depth.

It is necessary to recalculate the force per unit of the beam length in the reference state:

$$\underline{q}ds = \tilde{q}dx \Rightarrow \tilde{q} = \frac{q}{\dot{x}} = \underline{q}\sqrt{1+u'^2}. \quad (20)$$

Considering (19) and (20), in case of bending of a classical beam in a liquid, we come to the following statement for $x < l$:

$$\begin{aligned} a\partial^4 u = q = w + F[p_0 u'' + \lambda \varphi(u, u', u'')], \quad \lambda \rightarrow 0, \\ \varphi(u, u', u'') \equiv \gamma(uu'' + u'^2/2). \end{aligned} \quad (21)$$

Here $w \equiv mg - F\gamma$ is the beam weight per unit of length in a liquid (as in sec. 1); a formal small parameter λ was introduced here.

At the section of the fit with $x > l$, following outcomes of section 1 are valid:

$$u \equiv h, \quad \theta = 0, \quad Q = 0, \quad M = 0, \quad Q(l-0) = -R,$$

where R is the concentrated force, yet undetermined.

To solve equation (21), let us introduce a new variable and expand the length l in the form of an asymptotic series

$$0 \leq x \leq l = l_0(1 + \lambda l_1 + \dots); \quad x = l\zeta, \quad \partial_x = l^{-1}\partial_\zeta = l_0^{-1}(1 - \lambda l_1 + \dots)\partial_\zeta.$$

Then (21) could be rewritten in the following way:

$$al_0^{-4}(1 - 4\lambda l_1 + \dots)\partial_\zeta^4 u - Fp_0 l_0^{-2}(1 - 2\lambda l_1 + \dots)\partial_\zeta^2 u = w + \lambda F\varphi; \quad (22)$$

it is supplemented by the boundary conditions at the hinge and the continuity conditions

$$u(0) = 0, \quad M(0) = 0, \quad u(1) = h, \quad \theta(1) = 0, \quad M(1) = 0. \quad (23)$$

We will seek the solution of the (22)-(23) problem in the form of $u = u_0 + \lambda u_1 + \dots$ and come to a sequence of problems:

$$\begin{aligned} (\partial_\zeta^4 - k^2 \partial_\zeta^2)u_0 = wa^{-1}l_0^4 \equiv f_0, \quad u_0(0) = \partial_\zeta^2 u_0(0) = \partial_\zeta u_0(1) = \partial_\zeta^2 u_0(1) = 0, \quad u_0(1) = h; \\ (\partial_\zeta^4 - k^2 \partial_\zeta^2)u_1 = 2l_1(2\partial_\zeta^4 - k^2 \partial_\zeta^2)u_0 + F\varphi_0 a^{-1}l_0^4 \equiv f_1(\zeta), \quad k^2 \equiv Fp_0 a^{-1}l_0^2, \\ u_1(0) = \partial_\zeta^2 u_1(0) = u_1(1) = \partial_\zeta u_1(1) = \partial_\zeta^2 u_1(1) = 0. \end{aligned}$$

The general solution at the first step looks like

$$u_0 = A_0 \operatorname{ch} k\zeta + B_0 \operatorname{sh} k\zeta + C_0 + D_0 \zeta - \frac{f_0}{2k^2} \zeta^2.$$

Five conditions are sufficient to determine the constants A_0, \dots, D_0, l_0 . The general solution of the second problem is obtained by the Duhamel's integral:

$$u_1 = A_1 \operatorname{ch} k\zeta + B_1 \operatorname{sh} k\zeta + C_1 + D_1 \zeta + k^{-3} \int_0^\zeta f_1(\tau) \{ \operatorname{sh} k(\zeta - \tau) - k(\zeta - \tau) \} d\tau,$$

the small corrections u_1 and l_1 we will find on the second step.

5. Numerical calculation

Let us make a numerical calculation of a pipeline stress-strain state by the formulas received. We will assume that the pipeline's cross-section consists of two rings (Fig. 4), i.e., the internal steel and external concrete ones; their radiuses are $R_1 = 0.565$ m, $R_2 = 0.6$ m, $R_3 = 0.7$ m; these parameters correspond to a real Nord Stream Company's pipeline [10] which will be actually laid down on the Baltic Sea bottom. The properties of the steel are as following: Young's modulus $E_1 = 210$ GPa; Poisson's ratio $\nu_1 = 0.28$; density $\rho_1 = 7800$ kg/m³; properties of the class B20 concrete are: $E_2 = 27.5$ GPa; $\nu_2 = 0.2$; $\rho_2 = 2450$ kg/m³ [9]. Density of the sea water is $\rho_{liq} = 1028$ kg/m³.

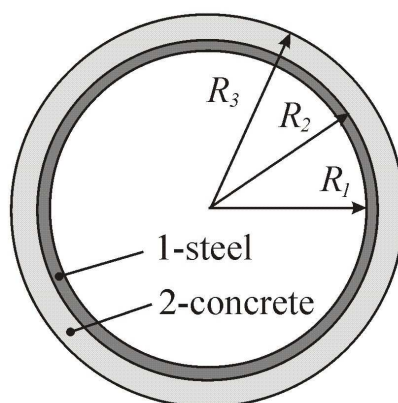


Fig. 4. Pipe cross-section

To find the bending and shear stiffness of a built-up beam, we will assume that the steel and concrete beams work in parallel; therefore, their joint stiffness is a sum:

$$a = a_1 + a_2, \quad a_n = E_n I_n, \quad I_n = \frac{\pi}{4} (R_{n+1}^4 - R_n^4),$$

$$b = b_1 + b_2, \quad b_n = \mu_n S_n N, \quad \mu_n = \frac{E_n}{2(1 + \nu_n)}, \quad n = 1, 2,$$

where I_n is the axial inertia moments of the cross-sections, S_n are their areas, μ_n are the shear moduli of the materials, and $N = 0.5$ is the shear ratio for a thin ring [1].

Equation (21) shows that significant divergences between the calculation results for beam deformations in linear statement in case of the new and commonly used accounting of the hydrostatic load should be expected under relatively high values of the pressure p_0 .

The deformed form of the beam is presented on Fig. 5; it has been found for three models: the classical and Timoshenko ones with simple accounting of the liquid's impact and the classical model with the hydrostatic load calculated by the new formula. The distance h is accepted equal to 50 m. The liquid pressure at the level of the hinge is $p_0 = 2$ MPa (corresponds to the depth of about 200 meters). Following values of the length of the beam's sagging part were received in accordance with section 4 formulas:

$$l_0 = 325.9 \text{ m}, l_1 = 0.05 \Rightarrow l = l_0(1 + l_1) = 342.5 \text{ m};$$

for the case of simple accounting of the liquid, we can find the values of l from (4) and (14): $l = 212.5$ m for the classical beam and $l = 211.6$ m for the Timoshenko beam.

As one can see from Fig. 5, displacements by classical model almost coincide with the displacements by Timoshenko model for the parameters chosen for the beam; therefore, further on, all dependences will be drawn only for the Bernoulli-Euler beam.

To find the contact pressure at the section of the fit by formula (5), let us calculate the weight of the beam in the liquid $w = 4.09$ kN and the concentrated forces: in case of simple accounting of the liquid $R = 0.43$ MN, by the new formula, $R = 0.21$ MN. For the Timoshenko model, we can find the external loading at the point of tangency from (15): $q_0 = -0.45$ MN and the exponent $\Lambda = 1.04 \text{ m}^{-1}$.

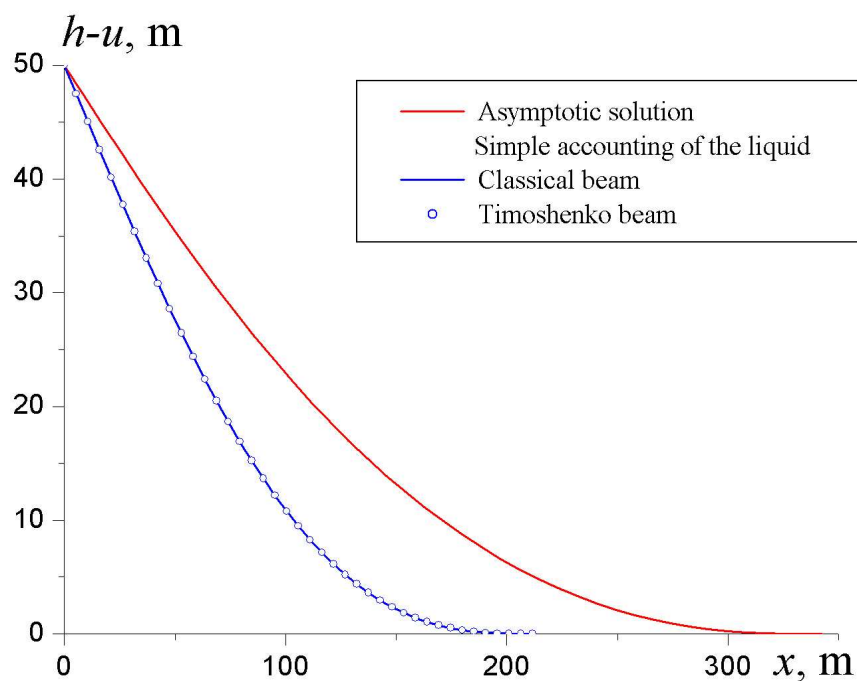


Fig. 5. The deformed form of the beam

Dependences of the shear force and bending moment at the beam's sagging part upon the value of parameter h are presented on Fig. 6; they were received by two approaches to accounting of the hydrostatic load. One can see from the plots that the maximal shear force occurs at the zone of fixing, and the divergence between the values received by the two approaches noticeably grows with the growth of h (it amounts to 52% for $h = 50$ m).

In case of the simple accounting of hydrostatic load, we will find that the maximal by absolute value bending moment always develops in the center of the beam's sagging part. Calculations under the new formula show that with growth of h , location of the maximum moves to the fixing point, while its value is much less. This way, for $h = 50$ m the maximal moment develops at the cross-section $x = 0.37l$ and the divergence in its value amounts to 66 %.

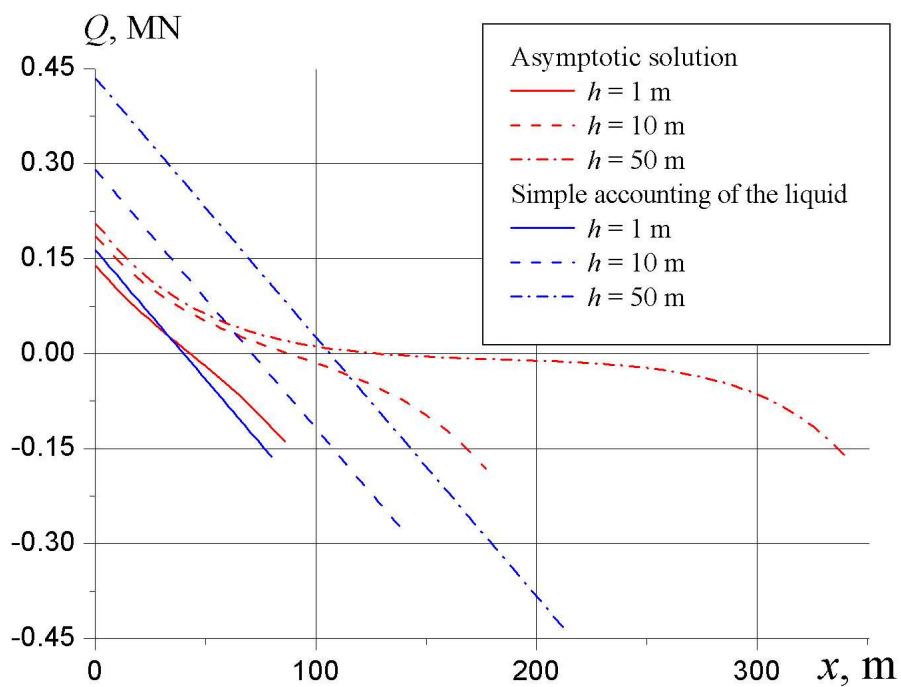
Dependences of the force and the moment on the magnitude of the liquid pressure p_0 at the level of the hinge are presented on Fig. 7. One can see that values of the bending moment and the shear force decrease at increasing of the liquid pressure. This effect cannot be predicted in case of simple account of the liquid. At big enough values of p_0 , the shear force turns zero far away from the ends of beam's sagging part, while the bending moment tends to a constant value:

$$M = -\frac{aw}{Fp_0}. \quad (24)$$

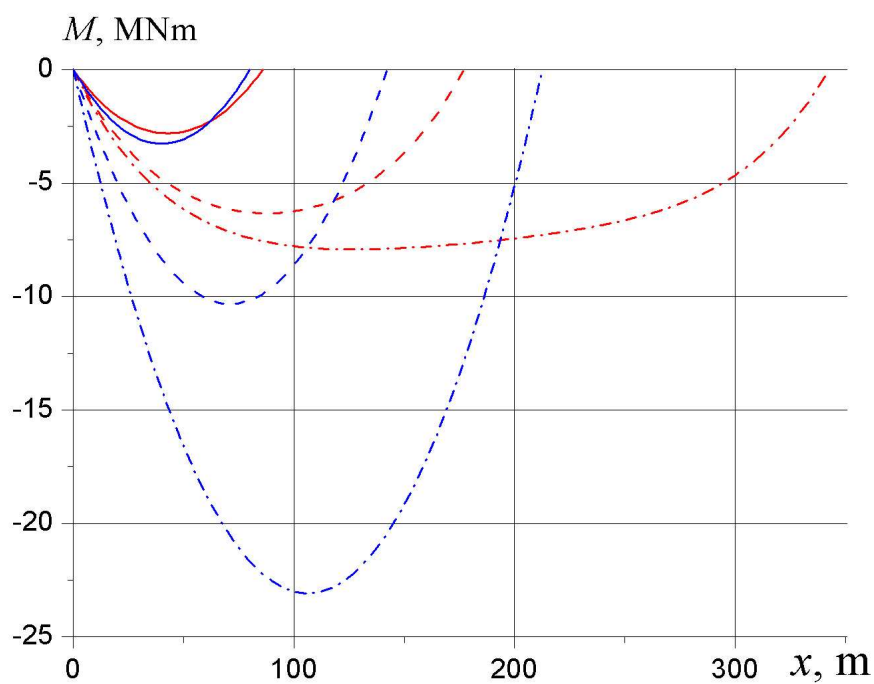
After determination of the bending moment M in a built-up beam, it is necessary to estimate arising stresses and to compare them with permissible stresses for concrete and steel. For this purpose, we will first find bending moments in each of the beams, and then apply an acknowledged engineering formula for calculation of maximal axial stress [2]:

$$M_n = \frac{Ma_n}{a}, \quad |\sigma_n^{\max}| = \frac{|M_n|}{I_n} R_{n+1}, \quad n = 1, 2. \quad (25)$$

As it is shown in [1], the asymptotic analysis of a three-dimensional problem of the theory of elasticity confirms validity of the hypothesis about the linear distribution of axial stresses at a beam's cross-section at its bending.

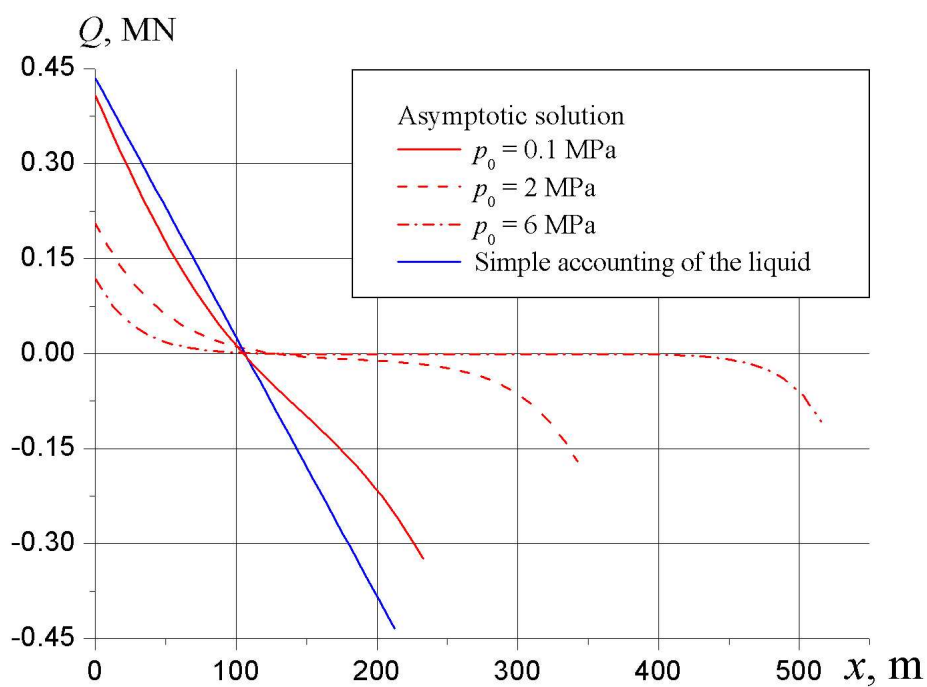


a)

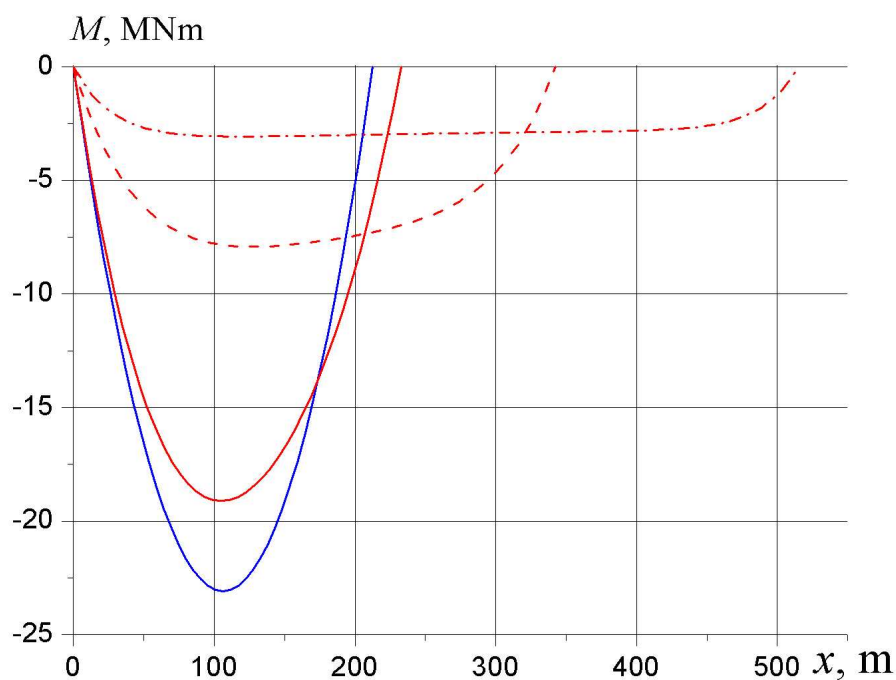


b)

Fig. 6. Shear force (a) and bending moment (b)
in the beam at various distances h



a)



b)

Fig. 7. Shear force (a) and bending moment (b)
in the beam at various pressures p_0

Dependence of the maximal pressure arising in a steel beam at the distance h and under pressure p_0 is presented on Fig. 8. In case of simple accounting of the liquid, we receive that the ultimate strength of steel $[\sigma_1]=400$ MPa is reached at $h=46$ m. With growth of pressure p_0 , the beam stresses go down and tend to a constant value non-dependent on h . At the distance $h=46$ m, the coefficient of safety for steel is equal $\eta=1.2$ at $p_0=0.1$ MPa and $\eta=7.3$ at $p_0=6$ MPa.

For real values of p_0 , the concrete tensile strength $[\sigma_2]=1.35$ MPa is considerably exceeded. It seems that it is necessary to reconsider modeling of the pipeline end fixing by using more complicated boundary conditions than the hinge.

In practice, for pipeline laying in the water from a vessel they use a stinger which is attached to the stern of the installation vessel and provides support to the pipeline [3]. The evaluated value of the rotation angle of the beam at the hinge gives an estimate of the angle necessary to fix the stinger depending on the depth of laying (Fig. 8). We should note that we have $\theta_0(0)=2h/l_0$ for the main term of the asymptotic series; a similar dependence follows from formulas (4).

6. Accounting of prestressing

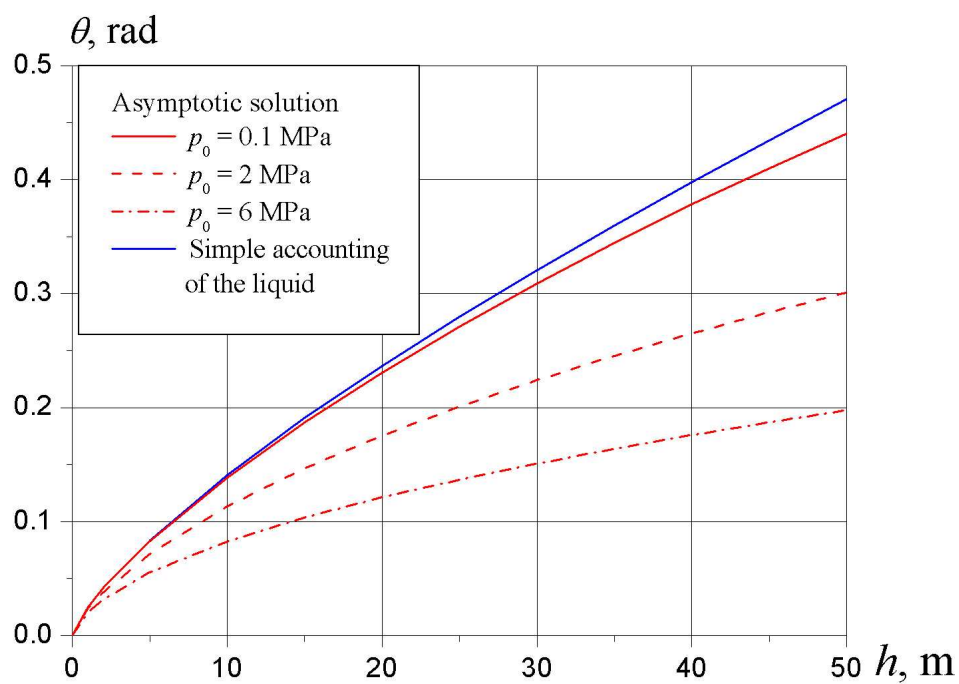
It is known that to decrease stresses arising in pipeline laying, the pipeline should be stretched. How big should this tension be depends on many factors and is an object of research.

The solution of section 4 could be easily generalized for this case. The fact is, the equation of bending of a beam preliminarily stretched by force T is given by

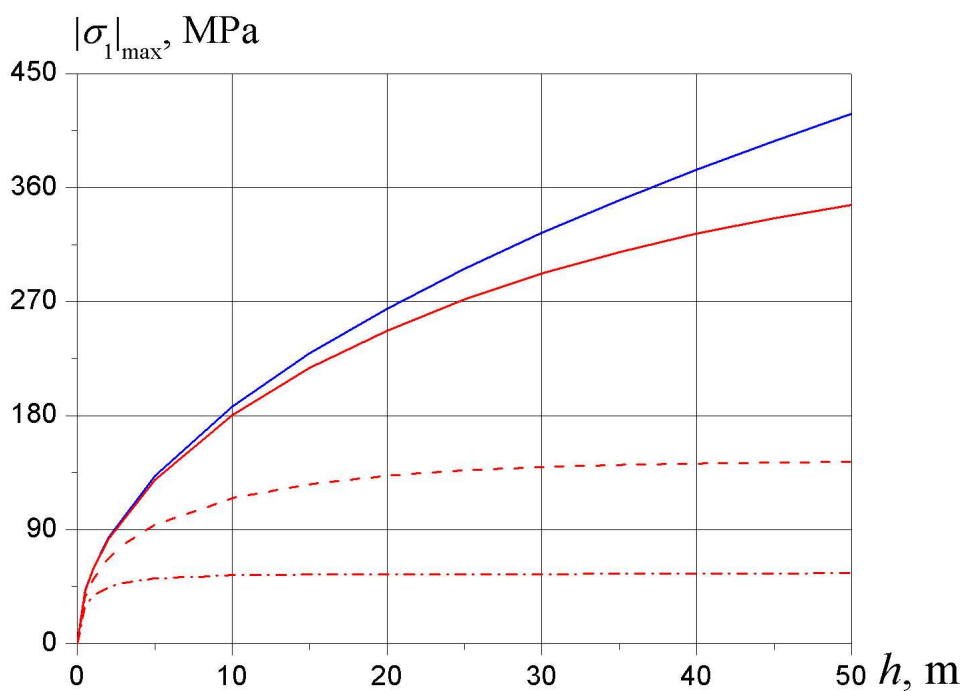
$$a\partial^4 u - Tu'' = q.$$

Considering hydrostatic loading (18) we can receive the statement (21) where we will have $\tilde{p}_0 \equiv p_0 + T/F$ instead of p_0 .

This way, accounting of prestressing of a pipeline is equivalent to increasing of the liquid pressure at the level of fixing (Fig. 7, 8).



a)



b)

Fig. 8. Dependences of the rotation angle at the hinge (a) and the maximal stress in steel (b) at the distance h and pressure p_0

Conclusion

The paper shows that the commonly known approach to accounting of hydrostatic loading of a beam as the Archimedes's force divided by the beam length leads to good enough results only for small displacements of an initially straight beam and at small depths. At bigger depths, big displacement, and initial curvature of the beam, it is necessary to use formula (18).

It is preferable to use the Timoshenko model for analysis of the contact pressure between the pipeline and the sea bottom.

Estimation of stresses arising in a pipeline has shown the necessity of obligatory modeling of the stinger that would allow to reduce loading on the pipeline at its laying down to permissible limits.

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