ANALITICAL MODEL FOR THE CALCULATION OF TEMPERATURE DISTRIBUTION IN THE OIL RESERVOIR DURING UNSTEADY FLUID INFLOW[1]

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This work represents an investigation of the temperature distribution caused by the barothermal effect during the single phase fluid inflow in the homogeneous oil porous reservoir to well with changing in time bottomhole pressure.

The mathematic model is developed, which is described the temperature changes in the reservoir with the pressure changing in time. It was made the comparison of calculation results with numerical solution.

The dependence between changing in time and space reservoir temperature and pressure during the steady fluid inflow was researched at first time by Checaluk E.B. in [1]. Later he had developed the method of the thermo analysis of the reservoir for constant flow rate, which is based on a registration in time of throttling temperature distribution of the fluid inflow. However, more easy way in the geophysics' researches is a case of the unsteady fluid inflow. For example, the pressure or temperature curves can be registries during well development or workover operations.

In this work it is represented a simple analytical model, which allows to calculate the temperature changes in the saturated porous formation at variable bottomhole pressure. The evidence of justifiability of our analytic model is made by the way of comparison of the received dependences and results of numerical problem solving.

Mathematical statement is given by the pressure transfer and the energy equation of Checaluk:

$$\begin{cases} \dfrac{\partial p}{\partial t} = \chi \Delta p \\ p\big|_{\vec{r}=W} = \varphi(t), p\big|_{t=0} = P_i, p\big|_{\vec{r}=G} = P_i, \end{cases} \quad (1)$$

$$\begin{cases} C_{res}\dfrac{\partial T}{\partial t} + C_{fl}\vec{v}[\nabla T + \varepsilon \nabla p] - \eta m C_{fl}\dfrac{\partial p}{\partial t} = \lambda_{res}\Delta T \\ T\big|_{t=0} = y(\vec{r}) \end{cases} \quad (2)$$

where

χ – reservoir pressure conductivity;

W и G – left and right reservoir board;

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C_{res}, C_{fl} – fluid and reservoir volume temperature capacity, $\frac{J}{kg \cdot K}$;

$\bar{v}$ - filtration rate vector, $\frac{m}{\sec}$;

η - adiabatic coefficient , $\frac{K}{Pa}$;

ε - throttling coefficient, $\frac{K}{Pa}$;

m – porosity;

λ_{res} - reservoir heat conductivity, $\frac{Wt}{m \cdot K}$.

The function $\varphi(t)$ is the dependence of well pressure on time. Initial pressure and temperature profile in the formation is equal, respectively, to P_i and $y(r)$.

Made assumptions for the receiving of analytical solution (1) and (2):

- hard reservoir model is fair: it is assumed that the fluid and skeleton compressibility are infinity small, i.e. $\beta^* = 0, \Rightarrow \chi = \frac{k}{\mu \beta^*} \to \infty$;
- thermal conductivity is absent;
- barothropic approximation is neglected by the influence of temperature change in the reservoir on the parameters of fluid and reservoir;
- temperature change is not influenced on the fluid and reservoir parameters;
- porous reservoir is homogeneous and horizontal.

Thereby the mathematical system (1) and (2) in cylindrical coordinates in the case of radial symmetric has the view:

$$\begin{cases} C_{res}\frac{\partial T}{\partial t} + C_{fl}v[\frac{\partial T}{\partial r} + \varepsilon\frac{\partial p}{\partial r}] - \eta m C_{fl}\frac{\partial p}{\partial t} = 0 \\ T|_{t=0} = y(r), \end{cases} \quad (3)$$

$$\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial p}{\partial r}) = 0 \\ p|_{r=r_w} = \varphi(t), p|_{t=0} = P_i, p|_{r=R_e} = P_i, \end{cases} \quad (4)$$

Pressure distribution. General solution (4) can be presented as $p(r) = C_2 + C_1 \ln(r)$. Considering the boundary conditions, we've got:

$$p(r,t) = P_i + \frac{(P_i - \varphi(t))}{\ln(\bar{R})} \ln(r / R_e), \text{ где } \bar{R} = R_i / r_w. \quad (5)$$

Temperature distribution. Let us divide (3) by C_{res}:

$$\frac{\partial T}{\partial t} + u[\frac{\partial T}{\partial r} + \varepsilon \frac{\partial p}{\partial r}] - \eta^* \frac{\partial p}{\partial t} = 0, \quad (6)$$

where $u(r,t) = \frac{C_{fl}}{C_{res}} v(r,t)$ - convection heat transfer rate, and $\eta^* = m \frac{C_{fl}}{C_{res}} \eta$.

The solution (6) along the special characteristic lines can be presented in view:

$$T(r,t) = y(r_1) + \varepsilon [p(r_1,0) - p(r_t,t)] + (\varepsilon + \eta^*) \int_0^t \frac{\partial p(r_\tau, \tau)}{\partial \tau}; \quad (7)$$

If we use the pressure expression (3) in temperature equation (7) then we can receive:

$$T(r_t,t) = y(r_1) + \varepsilon [p(r_1,0) - p(r_t,t)] - \frac{(\varepsilon + \eta^*)}{\ln(\bar{R})} \int_0^t \varphi'(\tau) \ln(\frac{r_\tau}{R_e}) d\tau; \quad (8)$$

The temperature expression (8) is presented by initial temperature profile and barothermal effect.

The characteristics are the solution of the following mathematic problem:

$$\begin{cases} \frac{dr}{dt} = u(r,t) = -\frac{k}{\mu} \frac{C_{fl}}{C_{res}} \frac{\partial p(r,t)}{\partial r} = -a \frac{P_{res} - \varphi(t)}{r} \\ r|_{t=0} = r_1, \end{cases} \quad (9)$$

where $a = \frac{k}{\mu \ln(\bar{R})} \frac{C_{fl}}{C_{res}}$.

Integration (10) results in:

$$r_t^2 = r_1^2 - 2a(P_i t - s(t)), \text{ where } s(t) = \int_0^t \varphi(\tau) d\tau, \; a = \frac{k}{\mu \ln(\bar{R})} \frac{C_{fl}}{C_{res}} \quad (11)$$

Thereby the equation for the change of the well temperature has a view:

$$T(r_w,t) = y(r_1) + \varepsilon [P_i - \varphi(t)] - \frac{(\varepsilon + \eta^*)}{\ln(\bar{R})} \int_0^t \varphi'(\tau) \ln(\frac{\sqrt{r_1^2 - 2a(P_i \tau - s(\tau))}}{R_e}) d\tau; \quad (12)$$

where $r_1 = \sqrt{r_w^2 + 2a(P_i t - s(t))}$.

On the fig.1 it is presented the temperature distribution: line 2 corresponds to the data of our analytical model and line 1 corresponds to the data of the numerical modeling [4]. In this case the well pressure is linearly decreased during two hours, i.e. it is described by the function:

$$\varphi(t) = \begin{cases} P_i - \dfrac{P_i - P_0}{\tau} t, \ if\ t < \tau \\ P_0, \ if\ t > \tau \end{cases}, \text{ where } P_i = 200 \text{ atm.}, P_0 = 100 \text{ atm.}, \tau = 2 \text{ hours.}$$

Fluid parameters	Value
density, kg/m^3	800
specific capacity, $\frac{J}{kg \cdot K}$	1800
viscosity, $Pa \cdot \sec$	0.003
throttling coefficient, K/Pa	4 10^{-7}
adiabatic coefficient, K/Pa	1.4 10^{-8}
Reservoir parameters:	
skeleton density, kg/m^3	2700
specific capacity, $\frac{J}{kg \cdot K}$	800
porosity	0.2
permeability, D	0.1

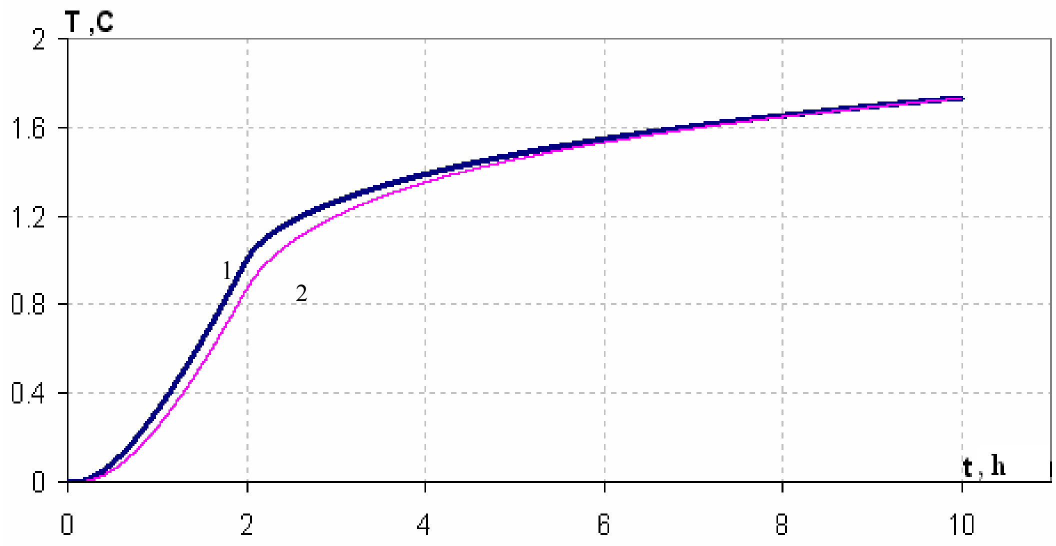

Figure 1. Comparison of the calculation results.

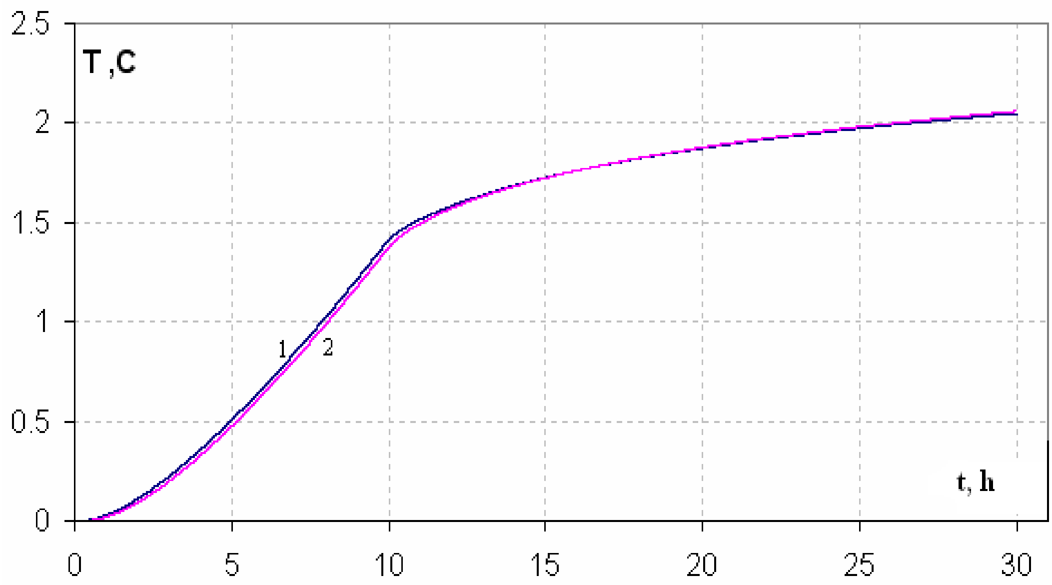

Figure 2. Comparison of the calculation results

On the fig. 2 it is shown the similar results but without considering the adiabatic effect and $\tau = 10\,\text{h}$. Line 1 corresponds to numerical solution and line 2 - to the analytical data.

This comparison shows the justifiability of our analytical model for describing the temperature distributions in the well with the variable bottomhole pressure.

Further approximations are connected with the approximated calculation of integral in the expression (12).

1. we can pull out the expression with logarithm using average time $0 \leq z \leq t$, then:

$$\int_0^t \frac{\varphi'(t)}{2\ln(\bar{R})} \ln\left(\frac{r_1^2 - 2a(P_i\tau - s(\tau))}{R_e^2}\right) d\tau = \ln\left(\frac{r_1^2 - 2a(P_i\tau - s(\tau))}{R_e^2}\right) \int_0^t \frac{\varphi'(t)}{2\ln(\bar{R})} d\tau = \frac{\varphi(t) - \varphi(0)}{2\ln(\bar{R})} \ln\left(\frac{r_1^2 - 2a(P_i z - s(z))}{R_e^2}\right), \quad (13a)$$

It is known that $\varphi(0) = P_i$ and using Euler's variables we've received:

$$\frac{\varphi(t) - \varphi(0)}{2\ln(\bar{R})} \ln\left(\frac{r_1^2 - 2a(P_i z - s(z))}{R_e^2}\right) = P_i - p\left(\sqrt{r^2 - 2a(P_i(z - t) - s(z) + s(t))}, t\right); \quad (13б)$$

Thereby the temperature equation has a view:

$$T(r,t) = y\left(\sqrt{r^2 + 2a(P_i t - s(t))}\right) + \varepsilon[P_i - p(r,t)] - (\varepsilon + \eta^*)[P_i - p\left(\sqrt{r^2 - 2a(P_i(z - t) - s(z) + s(t))}, t\right)], \quad (13)$$

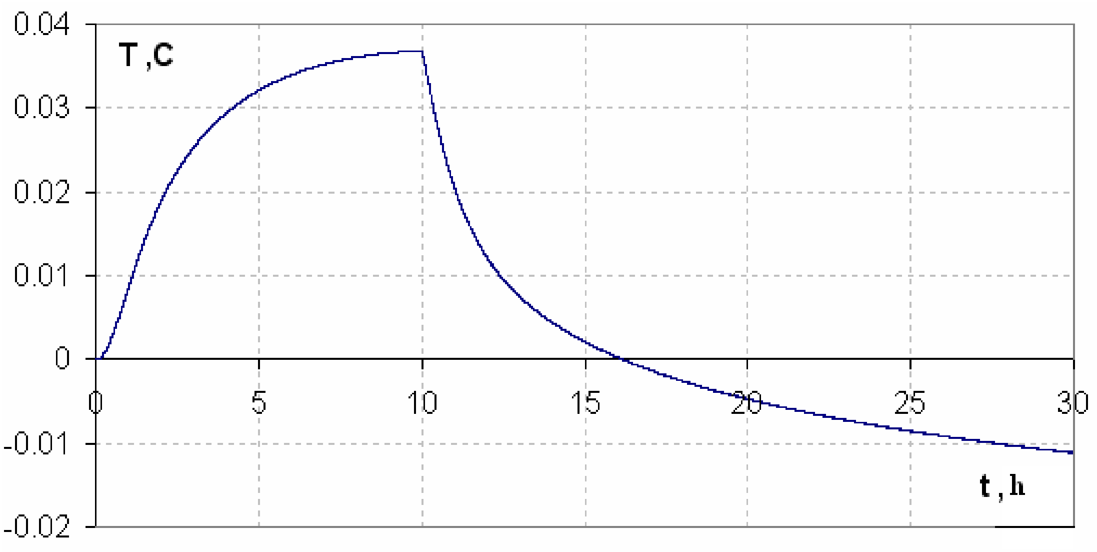

Figure 3. Temperature difference of our analytic model and the numerical model

2. In another case we'll take an integral by different parts:

$$\int_0^t \frac{\varphi'(\tau)}{2\ln(\bar{R})}\ln\left(\frac{r_1^2 - 2a(P_i\tau - s(\tau))}{R_e^2}\right)d\tau = \frac{\varphi(\tau) - P_i}{2\ln(\bar{R})}\ln\left(\frac{r_1^2 - 2a(P_i\tau - s(\tau))}{R_e^2}\right)\Big|_0^t - \\ - \int_0^t \frac{\varphi(\tau) - P_i}{2\ln(\bar{R})}\, d\ln\left(\frac{r_1^2 - 2a(P_i\tau - s(\tau))}{R_e^2}\right); \qquad (14a)$$

The second integral we'll calculate according to average theorem:

$$T(r,t) = y\left(\sqrt{r^2 + 2a(P_i t - s(t))}\right) + \varepsilon[P_i - p(r,t)] - (\varepsilon + \eta^*)[P_i - p(r,t) + p(r,z) - \\ - p\left(\sqrt{r^2 + 2a(P_i z - s(z))}, z\right)]; \qquad (14)$$

Thereby we'll receive approximate expressions for temperature distribution of the reservoir. Obviously, the accuracy of these formulas depends on the selection of optimal average values z.

On fig.4 and fig.5 it is shown the temperature curves, which were calculated using the equations (13) and (14) with different parameters of z that is noted by codes.

The fat curve corresponds to the temperature formula (12). The bottomhole pressure and model's parameters are similar to the values, which were described above.

On figures it is show that more optimal values for z are $z=\frac{t}{2}$ for equation (12) and z=t for equation (13).

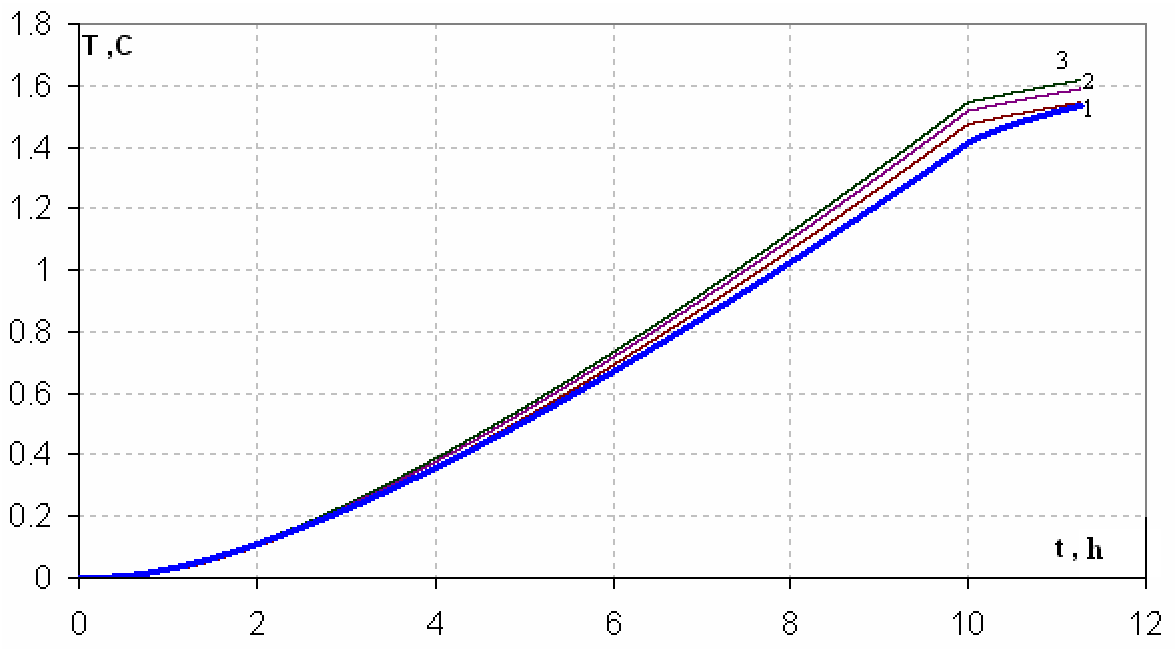

Figure 4. 1-3 – temperature curves according to formula (12)
with average parameter equal to $\frac{t}{2}, \frac{3t}{8}, \frac{t}{4}$.

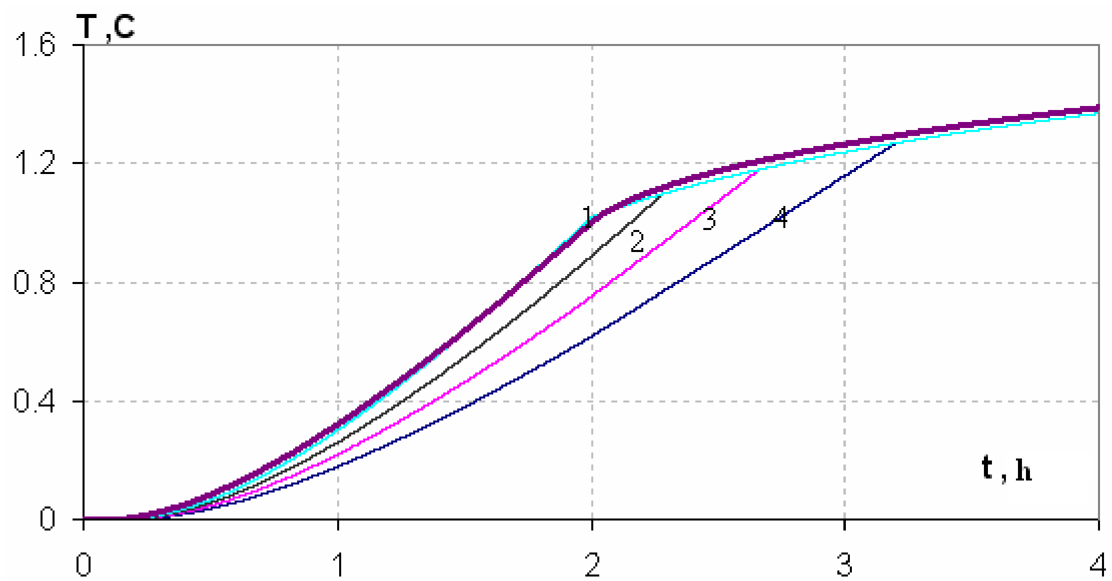

Figure 5**.** 1-4 – temperature curves according to formula (13)
with ave. parameter equal to t, $\frac{7t}{8}, \frac{6t}{8}, \frac{5t}{8}$.

Conclusion. Main purpose of this work is a receiving of simple analytical expressions for the calculation of temperature distribution in the well with variable bottomhole pressure. As a result:

- The mathematic model is developed, which is described the temperature changes in the reservoir with the pressure changing in time. It is developed the program-calculator for calculation of temperature distribution in reservoir and well.
- It was made the comparison of calculation results with numerical solution.

Analytic model, which is described in this work, was received for the single phase filtration of fluid in the homogeneous porous reservoir. However, it can be simply modified to model of non-homogeneous formation. Further, the developed model can be used for the solving the inverse problem of the definition of formation parameters according to the temperature change in the well.

References

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