THE METHODOLOGY OF PARAMETRIC OPTIMIZATION OF DETAILS AND DESIGNS OF COMPLEX CONFIGURATIONS

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The opportunity is substantiated and the algorithm of stage-by-stage consecutive parametric optimization of carrier details and designs of the complex configurations, based on correct application of strict mathematical methods is offered, serviceability and efficiency which is proved of many years practice of designing of the equipment of heavy mechanical engineering.

At designing heavily loaded carrier, foundation and case details or designs of the complex configurations, used in the equipment of rolling shops and subject to influence of technological and thermal loadings, inevitably there is a problem of a choice of the best, optimum from a position of this or that criterion of quality, parameters of details and regimes of cooling.

The statements and decisions of problems of optimum designing for real details enough complex, and frequently unique forms possess a series of features. The mathematical model making an information basis of statement anyone optimization of a problem, because of variety of designs of details should be enough universal, with opportunities of a wide variation boundary conditions, variables designing, area of change of managing parameters and restrictions as equalities or inequalities. To these qualities statement of the boundary-value problem, described by system of the differential equations in private derivatives with corresponding boundary and initial conditions to the greatest degree satisfies, one of which versions is the model of method of finite elements (MFE). Therefore the main difficulty of optimization of details and designs of complex configurations with a position of such widespread criterions of quality as the minimal (set) metal consumption, the maximal (set) rigidity or the minimal (set) intensity and so on, consists in high dimension and bulkiness of the mathematical models, describing their strained, deformed or thermoelastic states (TES), which can be in most cases submitted in the matrix form, as software of calculations on the computer of the stress-strain state (SSS) and TES. One more feature of the decision of such problems: exactness and reliability of the decision depend on a level information process of designing, i.e. completeness and accuracy of knowledge of conditions of load capacity, character of communications and degrees of freedom of a detail, thermophysics parameters, service properties of a material and so on, and this knowledge are rather approached and are limited. Therefore from the engineering point of view achievement mathematical a strict and exact optimum has no practical sense more often, as boundary and initial conditions of a boundary-value problem are never known with the big accuracy and always are available various, even mathematical not formalization, technological restrictions, “promoting” reception of only rational decision. Besides cost of calculation of one variant of a design, especially three-dimensional SSS or TES, can be very high, and, hence, the big practical importance complexity, so gets, and speed of convergence of algorithm of optimization. However nevertheless it is necessary to notice, that the opportunity of reception of the correct optimum decision is extremely important, as, first, this natural and primeval aspiration of the person to perfection and true and, secondly, at all not realizing it in a design, allows the engineer to define directions of reduction of weight of a detail at designing or at preservation of metal consumption to lower load capacity and to raise its uniformity so, to increase durability.

The analysis of the requirements showed to optimization of typical carrier details and constructions of the rolling equipment, allows to formulate the following kinds of mathematical statement of problems of optimization.

Let the elastic body, occupying area V in with border S is considered. As criterion of an optimality Y shall choose a maximum of equivalent stress $\forall \sigma_{equ}$ in researched area

$$Y(X) = \max_{z \in \overline{V}} \sigma equ(X,Z),$$

where Z - the vector of coordinates;

X - the vector of varied parameters of designing (the constructive sizes of elements),

$\overline{V}$ = VUS.

Problem 1. It is required to find a vector X*, which informs a minimum to criterion of an optimality

$$Y(X^*) = \min_{X} \max_{Z} \sigma equ\,(X,Z) \tag{1}$$

at restrictions such as equalities

$$\nabla\,(z,u,q) = 0, \qquad Z \in V; \tag{2}$$

$$\varphi(z,u,q) = 0, \ Z \in S \qquad (3)$$

and inequalities

$$\sigma equ(X^*, Z) \leq [\sigma], \quad \forall Z \in V; \qquad (4)$$

$$/ui(X^*,Z)/ \leq [u], \quad \forall Z \in V, \ i = 1,2,3; \qquad (5)$$

$$X^* \in X, \qquad (6)$$

where ∇ - the differential operator, determining a regional problem of the theory of elasticity;

φ - the operator, assigning boundary conditions;

u - a vector of displacements;

q - a vector of loadings;

X - the permissible set of varied parameters, determined by geometrical, constructive and technological conditions;

[σ]- the condition of strength, dependent on a material of a design, a kind of a strained state, character of a loading of the accepted settlement scheme and other factors;

[u] - the top border of restrictions of the displacements, dependent on permissible values of the clearances between the details, required rigidity, etc.

However for some details, for example such as the housing of sheet hot or cold rolling mill, the determining parameter is rigidity. Therefore as goal function it is necessary accept a level of displacements of ui. Then a problem of optimization can to be put as follows.

Problem 2. To find a vector X*, which informs a minimum to criterion of an optimality

$$Y(X^*) = \min_{X} \max_{Z \in \overline{V}} /u_i(X, Z)/$$

at presence of restrictions such as equality (2), (3) and inequalities (4), (5), and also i accepts value 1, either 2 or 3.

Except for problems 1 and 2, for series of details it is necessary to allocate also a problem of minimization of mass, which can be formulated as follows:

Problem 3. To find vector X*, which informs a minimum to criterion of an optimality

$$Y(X^*) = \min_{X} \Omega(X) = \min_{X} \int_{V(X)} \gamma(Z)\,dV,$$

where γ - density of a material, at presence of restrictions (2) - (4).

The given mathematical statements demonstrate the approach to problems of optimization by a principle of "a black box", when the condition of object is described by two groups of parameters: entrance (independent) variables of designing (varied parameters of a design is a vector X) and exit (dependent) parameters of quality of functioning of object of Y. Such the approach is simple and convenient for the majority of real monolithic and welded designs, when optimization is preceded, as a rule, with alternative calculations and the careful analysis of the SSS or TES. Application of this principle for parametrical optimization of designs, in which calculation of the SSS is not so expensive also quantity of parameters of designing is limited , and it is required to model set of variants of restrictions and conditions loading (for example, for search of critical conditions of destruction) is especially practical.

The choice of an effective method of the decision of a problem of optimization depends on features of change of the SSS of a detail at a variation of its parameters. By optimization of the form for models of behaviour of designs based on MFE, there is big freedom of a choice of the varied parameters, determining statement and efficiency of a problem. But even at discrete nodular representation of flat model MFE of a stress is necessary to define in several hundreds points. Usually at the analysis of the SSS are limited to consideration of the points, lying on a contour of a detail, but also in this case their number reaches several tens. It troubles the analysis of results of calculation. Therefore is expedient to build criterion function for the characteristic points, taking into account features of the SSS or TES all detail, for example, for points of a maximum of intensity or a maximum of equivalent stresses.

Thus, for the majority of problems of optimization at designing details of a complex configuration the following features are characteristic:

1. As a rule, communication between criterion of an optimality Y and a vector X has not obvious character, and is carried out through system of the differential equations of the second order.

2. The mathematical model of optimized object is expressed not in an obvious analytical kind, and in the form of the operator, realized as software of calculations of the SSS or TES on a computer.

3. The big dimension of space of optimized parameters.

4. Significant expenses of machine time for calculation of one value of criterion function.

5. The mixed composition of restrictions on characteristics of object, i.e. at mathematical statement there are both equality and inequalities.

At the decision of such complex problems of optimization strict mathematical methods arise significant difficulties. However to their decision is possible to apply a combination of known methods: planning of computing experiments and the decision of problems of mathematical programming. The method of planning of experiments allows to execute approximation of the numerical decision : to find obvious dependence for criterion function and functions of restrictions, to estimate substantiality of influence of factors on criterion of optimization in various ranges of change of managing parameters and to reduce quantity of calculation variants. Then the problem of optimization is easily to reduce to a problem of linear or nonlinear programming, applying for the decision, for example, a simplex - method of the Nelder - Mid or a method of the quickest descent.

The decision of a problem of parametrical optimization at designing details of complex configurations from a position of this or that criterion of quality is offered to be carried out stage by stage in the following sequence (see fig. 1):

1. To develop or accept mathematical model of an optimized detail, which is represented as software of calculations of the SSS or TES on the computer. Further this model to consider as " a black box ". On its entry varied parameters move, and on an output parameters of the SSS or TES object of designing are removed.

2. With use of the accepted model after a series of trial calculations to reveal variables of designing (design parameters) most essentially influencing on a pressure and a deformations. If in result the configuration of the detail meeting the set requirements (for example, to a level of pressure, metal consumption, etc.) is determined, calculations can be stopped.

3. If the variant of a design does not meet the set requirements , that on the revealed variables of designing, most essentially influencing on exit parameters of a detail, to plan computing experiment.

4. To execute a series of calculations under the plan of full factorial experiment, thus the maximal number of variants equally 2^n, where n-quantity of parameters varied at two levels; it is possible to apply to decrease of variability and fractional factorial experiment.

5. In an environs of an initial point (a projected detail corresponding to the prototype) on the basis of statistical methods: the software of correlation and plural regression the analysis of a matrix of planning to execute local approximation of characteristics of system (functions of the response) the linear or nonlinear equations of regress, representing to dependence of the maximal values of stresses and deformations in characteristic points of a detail from its design parameters and their pair interactions.

6. To execute analysis the received obvious dependence (the equations of regress) for goal functions of restrictions with revealing most statistically significant design parameters. If the received information has not enough for decision-making and designing of a rational design to proceed to the following stage.

7. Using strict methods of the decision of problems of mathematical programming, at presence of system of restrictions on variables of designing and a level of stresses and deformations in a series of points, and also accepting goal function (a level of stresses, metal consumption, etc.) to receive a rational (optimum) variant of a design, that is carried out effectively enough at small expenses of computer times.

For realization of the submitted algorithm (see fig. 1) optimization of designs had been developed the corresponding software.

The formed algorithm of the stage-by-stage, consecutive decision of a problem of parametric optimization of complex designs possesses a number of the advantages confirmed in practice [1, 2]:

1) association and close interaction of programs of optimization with the software for researches of the SSS or TES designs, that provides fast convergence and economically of algorithm;

2) the account of a significant share of experience, intuition and common sense of the qualified design engineer and his active participation in decision-making process;

3) Formed on a stage No 5 equations of regress, approximating communication of the SSS in characteristic points with variables of designing, can be used directly in calculation practice at designing for the analysis of the SSS or TES and a choice of rational parameters without attraction of a computer, and also by development CAD carrier and thermo-loaded details, construction of their parametric series and for unification; besides, the equations of regress allow to estimate sensitivity of criterion of an optimality to parameters (to define gradients at movement to an optimum) and to choose the best design decision;

4) An opportunity of addition with the known software, realizing in real time a dialogue regime of correction of all components of boundary conditions, including and configurations of a detail, that to a considerable degree approaches process of optimization to the organization of the closed system of the automated synthesis of optimum constructions.

Performance capability and efficiency of the created algorithm of consecutive parametric optimization and its the software are proved convincingly on numerous examples of improvement of designs of housings of blooming mills, of continuous - billet and sheet hot and cold rolling mills, of universal joints and rods of various designs, of typical carrier details and of designs mining, drilling and other equipment of heavy mechanical engineering [1, 2].

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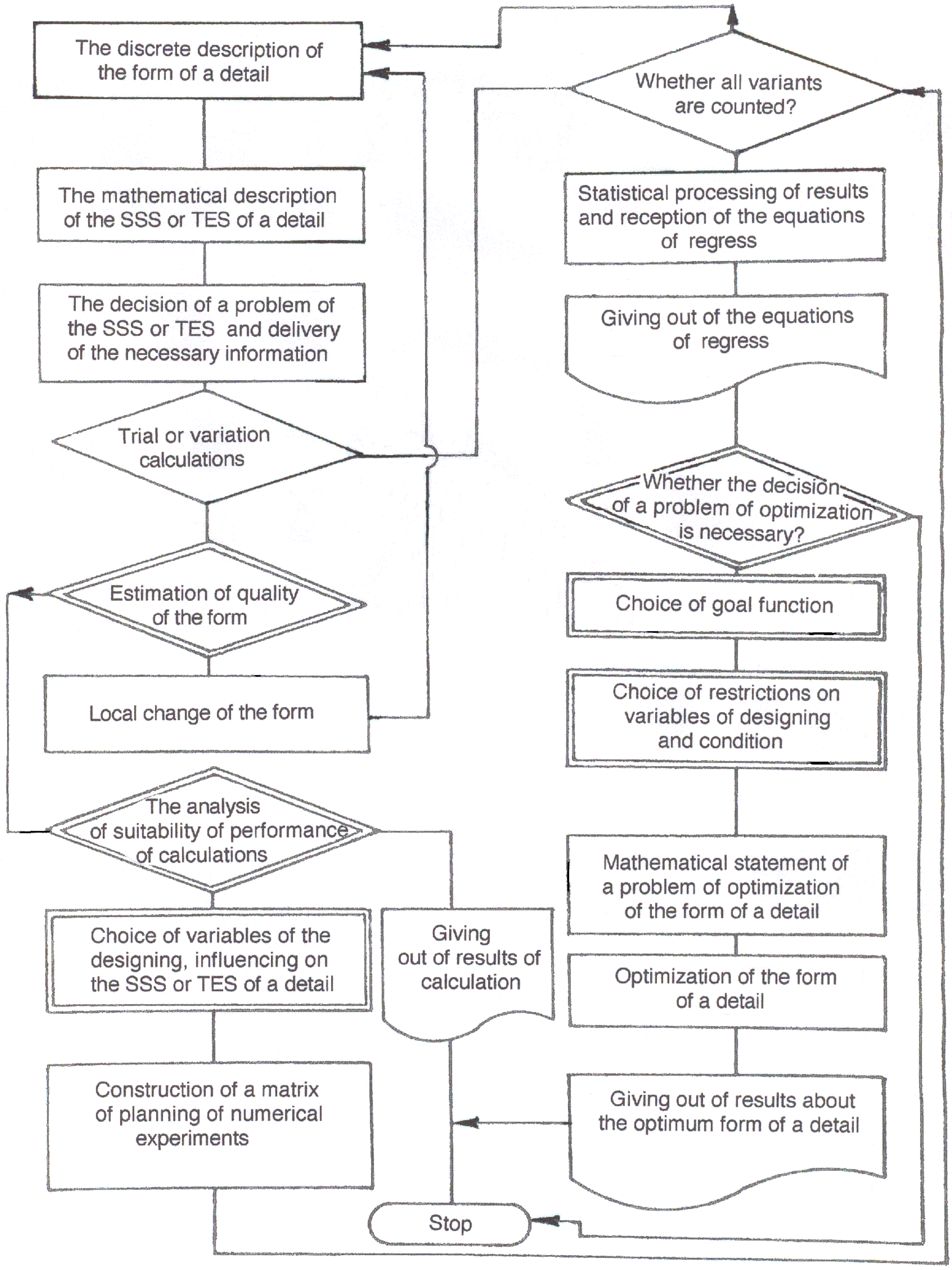

Figure 1. The block-scheme of algorithm of the stage-by-stage decision of a problem of multi variant designing of optimum details of a complex configuration